CS101 Midterm Exam

Fall 2007

October 15, 2007

I. Search

1. Terms and Definitions.

Define in your own words the following terms associated with search:

1. state:
2. state space:
3. search tree:
4. search node:
5. goal:
6. action:
7. successor function:
8. branching factor:

2. Search.

Suppose there are two friends living in different cities on a map, such as the Romania map in the textbook. On every turn, we can move each friend simultaneously to a neighboring city on the map. The amount of time needed to move from city i to neighbor j is equal to the road distance $d(i, j)$ between the cities, but on each turn the friend that arrives first must wait until the other one arrives (and calls the first on his/her cell phone) before the next turn can begin. We want the two friends to meet as quickly as possible. Let us formulate this as a search problem.

(a) What is the state space? (You will find it helpful to define some formal notation here.)

(b) What is the successor function?

(c) What is the goal?

(d) Let $SLD(i, j)$ be the straight-line distance between any two cities i and j. Which, if any, of the following heuristic functions are admissible?(If none, write NONE.)
(i) $SLD(i, j)$
(ii) $2 \times SLD(i, j)$
(iii) $SLD(i, j)/2$

(e) True/False: There are completely connected maps for which no solution exists.

4. Constraint Satisfaction Problems

A constraint satisfaction problem (or CSP) is defined by a set of variables, $x_1, x_2, \ldots, x_n$, and a set of constraints, $c_1, c_2, \ldots, c_m$. Each variable x_i has a nonempty domain D_i of possible values. Each constraint c_i involves some subset of the variables and specifies the allowable combinations of values for that subset. A state of the problem is defined by an assignment of values to some or all of the variables. Now, give precise formulations for each as constraint satisfaction problems:

1. Rectilinear floor planning: find non-overlapping places in a large rectangle for a number of smaller rectangles.
2. Class scheduling: There is a fixed number of professors and classrooms, a list of classes to be offered, and a list of possible time slots for classes. Each professor has a set of classes that he or she can teach.

II. Logic and Reasoning

1. Natural Deduction

1. Prove $(p \wedge q) \wedge r$, $s \wedge t \vdash q \wedge s$
2. Prove $(p \rightarrow r) \wedge (q \rightarrow r) \vdash \ (p \wedge q) \rightarrow r$

2. Translations from English to First-order logic

1. No man is both a butcher and a vegetarian.
2. All men except butchers like vegetarians.
3. The only vegetarian butchers are women.
4. No man likes a woman who is a vegetarian.

3. Translations to Conjunctive Clausal Form

A formula is in conjunctive normal form (CNF) if it is a conjunction of clauses, where a clause is a disjunction of literals. For example, all of the following formulas are in CNF: $A \wedge B$, $\neg A \wedge (B \vee C)$, and $(A \vee B) \wedge (\neg B \vee C)$.

1. $\forall x \forall y [P(x, y) \rightarrow [Q(x, y) \rightarrow R(x, y)]]$
2. $\neg \forall x \exists y [P(x, y) \rightarrow Q(x, y)]$
3. $\forall x \forall y [[P(x, y) \vee Q(x, y)] \rightarrow R(x, y)]$

4. Unification

For each pair, give the substitution that unifies the two expressions:

1. Color(Tweety,Yellow), Colory(x,y)
2. Color(Hat(Mailman),Blue), Color(Hat(y),x)
3. R(F(x),B), R(y,z)
4. R(F(y),x), R(x,F(B))

Extra Credit 1.

Solve by resolution to show that I win.

> Heads I win, tails you lose.

Here are the axioms and expression to prove, where H is "heads", W is "I win", T is "tails", and L is "You lose".

1. $H \rightarrow W, T \rightarrow L, T \vee H, L \rightarrow W \ \vdash \ W$

Extra Credit 2.

Translate the following from First-order logic to English.

1. $\forall x[hesitates(x) \rightarrow lost(x)]$
2. $\neg\exists x[business(x) \wedge is - like(x, Showbusiness)]$
3. $\exists x \forall t[Person(x) \wedge time(t) \wedge Can - fool(x, t)]$

Appendix: Strategies for Natural Deductive Proofs

Given a set of premises, Δ, and the goal you are trying to prove, Γ, there are some simple rules of thumb that will be helpful in finding ND proofs for problems. These are not complete, but will get you through your problems. Here goes:

> Given Δ, prove Γ:

1. Apply premises, $p_i \in \Delta$, to prove Γ.
2. If you need to use a $\vee$-premise, then apply $\vee$-e and prove Γ for each disjunct.
3. Otherwise, work backwards from the type of goal you are proving:
   (a) If the goal Γ is a conditional ($A \rightarrow B$), then assume A and prove B; use $\rightarrow$-i rule.
   (b) If the goal Γ is a a negative ($\neg A$), then assume $\neg\neg A$ and prove contradiction; use $\neg$-i rule.
   (c) If the goal Γ is a conjunction ($A \wedge B$), then prove A and prove B; use $\wedge$-i rule.
   (d) If the goal Γ is a disjunction ($A \vee B$), then prove one of A or B; use $\vee$-i rule.