Problem Set 1 - Part 1

cs112

Handed Out: Tue. September 14, 2004
Due Date: Fri. September 24, 2004 – Due in class on paper

Exercise 1 Propositional Logic

Part 1.1 Tautologies, Contradictions, and Contingencies

Let ϕ be a tautology, ψ be a contradiction, and χ be a contingency. Which of the following sentences are (i) tautological, (ii) contradictory (unsatisfiable), or (iii) contingent (satisfiable).

1. $\phi \wedge \chi$
2. $\phi \vee \chi$
3. $\psi \wedge \chi$
4. $\psi \vee \chi$
5. $\phi \vee \psi$
6. $\chi \rightarrow \psi$

Part 1.2 Propositional Logic Natural Deduction

Give a formal proof of validity for each of the following sequents. (A conditional or indirect proof will be much easier in some.) Useful information can be found in:

- `http://ilab.usc.edu/classes/2002cs561/notes/session16-18.pdf`
- `http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/logic/logicintro.html` (sections 4 to 6)

1. $\vdash (p \wedge q) \rightarrow p$
2. $p \rightarrow q,\ q \rightarrow r,\ \neg r \vdash \neg p$
3. $p \rightarrow \neg q,\ r \rightarrow q,\ \neg r \rightarrow s \vdash p \rightarrow s$
4. $\neg p \rightarrow q,\ r \rightarrow (s \vee t),\ s \rightarrow \neg r,\ p \rightarrow \neg t \vdash r \rightarrow q$
5. $p \rightarrow (q \wedge r),\ q \rightarrow s,\ r \rightarrow t,\ (s \wedge t) \rightarrow \neg u,\ u \vdash \neg p$
6. $\neg p \vee \neg q \vdash \neg(p \wedge q)$
7. $p \wedge q \vdash \neg(\neg p \wedge \neg q)$

Part 1.3 Translation into CNF

Translate the following into Conjunctive Normal Form.

1. $(\neg p \wedge q) \rightarrow (p \wedge (r \rightarrow q))$
2. $p \rightarrow ((q \rightarrow r) \rightarrow (\neg r \vee q))$
3. $(\neg p \rightarrow q) \wedge ((\neg(\neg q \rightarrow \neg r) \vee r))$

Part 1.4 Propositional Logic Resolution

Use resolution to show that the following list of clauses is unsatisfiable.

```
{P, Q}
{Q, R}
{R, W}
{-R, -P}
{-W, -Q}
{-Q, -R}
```

Exercise 2 Predicate Calculus

Part 2.1 Translation

Translate the following sentences into predicate logic. Give the key for the variables and predicate letters that you choose. If you think that more than one translation is suitable, give the alternatives and discuss their differences. Represent as much as possible of the structure relevant to quantificational arguments.

(a) Everything is black or white.

(b) Either everything is black or it is white.

(c) A girafe is a quadruped.

(d) Everybody loves somebody.

(e) There is someone whom everyone loves.

(f) If you love a woman, kiss her or lose her.

(g) If no one kisses John, Mary will.

(h) People who live in New York love it.

(i) If John does not love New York, he does not live there (i.e., in it).

(j) If someone does not love New York, he does not know it.

Part 2.2 Predicate Calculus Equivalences

Each part of this exercise consists of an English sentence followed by a translation of it in predicate logic and a number of additional formulas. Indicate which of the formulas are equivalent to the translation and give the laws or rules necessary to show this equivalence (the appendix might be helpful for this task). There may be more than one equivalent formulas.

(a) Everything has a father and all odd numbers are integers.
$(\forall x)(\exists y)F(y,x) \wedge (\forall z)(O(z) \rightarrow I(z))$

(1) $(\forall z)(\forall x)(\exists y)(F(y,x) \wedge (O(z) \rightarrow I(z)))$
(2) $(\forall z)(\exists y)(\forall x)(F(y,x) \wedge (O(z) \rightarrow I(z)))$
(3) $(\forall x)(\forall z)(\exists y)(F(y,x) \wedge (O(z) \rightarrow I(z)))$

(b) If adam is a bachelor, then not all men are husbands.
$B(a) \rightarrow \neg(\forall x)(M(x) \rightarrow H(x))$

(1) $(\forall x)(B(a) \rightarrow \neg(M(x) \rightarrow H(x)))$
(2) $(\exists x)(B(a) \rightarrow \neg(M(x) \rightarrow H(x)))$
(3) $\neg(B(a) \rightarrow (\forall x)(M(x) \rightarrow H(x)))$
(4) $B(a) \rightarrow (\exists x)(M(x) \wedge \neg H(x))$

Part 2.3 First-Order Logic Natural Deduction

Prove the validity of the following sequents. The information provided in the Appendix and the links given above may be of help to you.

1. $\neg(\exists x)(P(x) \wedge Q(x)),\ (\exists x)(P(x) \wedge R(x)) \ \vdash\ (\exists x)(R(x) \wedge \neg Q(x))$
2. $(\forall x)(P(x) \rightarrow Q(x)),\ P(a),\ R(a) \ \vdash\ (\exists x)(R(x) \wedge Q(x))$
3. $(\forall x)((P(x) \vee Q(x)) \rightarrow R(x)),\ (\forall x)((R(x) \vee S(x)) \rightarrow T(x)) \ \vdash\ (\forall x)(P(x) \rightarrow T(x))$
4. $(\exists x)(\forall y)(P(x,y)) \ \vdash\ (\forall y)(\exists x)(P(x,y))$

Appendix

Below are some inference rules. These are equivalent to those included in part 2 of the handout, but with a different notation.

Universal Instantiation

$$\begin{array}{ll} & (\forall x)\phi(x) \\ \hline \therefore & \phi(c) \end{array}$$

Existential Generalization

$$\begin{array}{ll} & \phi(c) \\ \hline \therefore & (\exists x)\phi(x) \end{array}$$

Laws of Quantifier Negation

Law 1: $\neg(\forall x)\phi(x) \Leftrightarrow (\exists x)\neg\phi(x)$
Law 2: $(\forall x)\phi(x) \Leftrightarrow \neg(\exists x)\neg\phi(x)$
Law 3: $\neg(\forall x)\neg\phi(x) \Leftrightarrow (\exists x)\phi(x)$
Law 4: $(\forall x)\neg\phi(x) \Leftrightarrow \neg(\exists x)\phi(x)$

Laws of Quantifier In(Dependence)

Law 5: $(\forall x)(\forall y)\varphi(x,y) \Leftrightarrow (\forall y)(\forall x)\varphi(x,y)$
Law 6: $(\exists x)(\exists y)\varphi(x,y) \Leftrightarrow (\exists y)(\exists x)\varphi(x,y)$
Law 7: $(\exists x)(\forall y)\varphi(x,y) \Rightarrow (\forall y)(\exists x)\varphi(x,y)$

Laws of Quantifier Movement

Law 9: $(\varphi \rightarrow (\forall x)\psi(x)) \Leftrightarrow (\forall x)(\varphi \rightarrow \psi(x))$ **provided that** x **is not free in** φ
Law 10: $(\varphi \rightarrow (\exists x)\psi(x)) \Leftrightarrow (\exists x)(\varphi \rightarrow \psi(x)$ **provided that** x **is not free in** φ
Law 11: $(\forall x)\varphi(x) \rightarrow \psi \Leftrightarrow (\exists x)((\varphi)(x) \rightarrow \psi)$ **provided that** x **is not free in** ψ
Law 12: $(\exists x)\varphi(x) \rightarrow \psi \Leftrightarrow (\forall x)((\varphi)(x) \rightarrow \psi)$ **provided that** x **is not free in** ψ