CS112 Notes
Temporal Logic: Part I

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1 Propositional Tense Logic

We first look at a simple but powerful extension to propositional logic (and later, for a restricted subset of predicate logic) where propositions, ϕ_i, are *tensed*, and are interpreted relative to a model of time in additional to the other parameters of interpretation that we've discussed thus far.

We will introduce four operators over propositions, attributed to Prior (1967) and described below:

(i) $\mathbf{G}\phi$: it is always going to be the case that ϕ

(ii) $\mathbf{H}\phi$: it always has been the case that ϕ

(iii) $\mathbf{F}\phi$: it will at some point in the future be the case that ϕ

(iv) $\mathbf{P}\phi$: it was at some point in the past the case that ϕ

F and **P** are usually referred to as *weak operators*. They can be defined in terms of the other two as follows: $\mathbf{F}\phi = \neg\mathbf{G}\neg\phi$; $\mathbf{P}\phi = \neg\mathbf{H}\neg\phi$; These operators together with the syntax and combinatorics of propositional logic give us propositional tense logic (PTL).

To get a feeling for how PTL works, consider the following expressions, as described by Prior himself:

(a) $\mathbf{G}\phi \rightarrow \mathbf{F}\phi$: What will always be, will be;

(b $\mathbf{G}(\phi \rightarrow \psi) \rightarrow (\mathbf{G}\phi \rightarrow \mathbf{G}\psi)$: If ϕ will always imply ψ, then if ϕ will always be the case, so will ψ.

(c) $\mathbf{F}\ \phi \rightarrow \mathbf{F}\ \mathbf{F}\phi$: If it will be the case that ϕ, then it will be —-in between— that it will be.

1.1 Minimal Tense Logic

The system K_t generated by the following four axioms:

(a) $\phi \rightarrow \mathbf{H}\ \mathbf{F}\phi$: What is, has always been going to be;

(b $\phi \rightarrow \mathbf{G}\ \mathbf{P}\phi$: What is, will always have been;

(c) $\mathbf{H}(\phi \rightarrow \psi) \rightarrow (\mathbf{H}\phi \rightarrow \mathbf{H}\psi)$: Whatever always follows from what always has been, always has been;

(d) $\mathbf{G}(\phi \rightarrow \psi) \rightarrow (\mathbf{G}\phi \rightarrow \mathbf{G}\psi)$: Whatever always follows from what always will be, always will be.

Add to this the following two rules of temporal inference below in order to prove temporal propositional expressions.

(i) **Rule H (RH)**: From a proof of ϕ, derive a proof of $H\phi$.

(ii) **Rule G (RG)**: From a proof of ϕ, derive a proof of $G\phi$.

1.2 A Model for PTL

A model for PTL, **M**, can be defined. Let T be a set of moments in time, and let R be the *earlier than* relation.

(i) $V_{M,t}(\mathbf{G}\phi) = 1$ iff for every $t_i \in W$ such that tRt_i: $V_{M,t_i}(\phi) = 1$

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(i) $V_{M,t}(\mathbf{P}\phi) = 1$ iff for some $t_i \in W$ such that t_iRt: $V_{M,t_i}(\phi) = 1$