A study in portfolio management

Magnús Eðvald Björnsson
magnus@cs.brandeis.edu

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1 Introduction

In this paper I don't assume the reader has extensive knowledge of stock markets and the portfolio selection problem. It is therefore a good idea to start with a general introduction to general portfolio theory.

The seminal work developing the modern portfolio theory is credited to Harry Markowitz (see [6]), co-winner of the 1990 Nobel prize in economics. Markowitz's approach begins by assuming that an investor has a given sum of money to invest at the present time. At the end of the *holding period* (the length of time the money will be invested), the investor will sell the securities that were purchased at the beginning of the period and then either spend the proceeds on consumption or reinvest the proceeds in various securities. At the beginning the investor must make a decision on what particular securities to purchase and hold until the end of the period. Because a portfolio is a collection of securities, this decision is equivalent to selecting an optimal portfolio from a set of possible portfolios, often referred to as the *portfolio selection problem.*

Understanding the portfolio problem as a decision problem under risk, showed to be extremely fruitful. Subsequently, Sharpe, Lintner and Mossin developed the *Capital Asset Pricing Model (CAPM)* which represents the core of the modern capital market theory (see [4]).

2 On Portfolio Management

Before diving head first into Information Theoretical aspects of portfolio selection, an introduction to some basic concepts from elementary Investment Theory is in order.

2.1 Indifference Curves and Risk Aversion

The method used in selecting the most desirable portfolio involves the use of *indifference curves.* These curves represent an investor's preferences for risk and return. It can be drawn on a two-dimensional graph, where the horizontal axis usually indicates risk as measured by variance or standard deviation and the

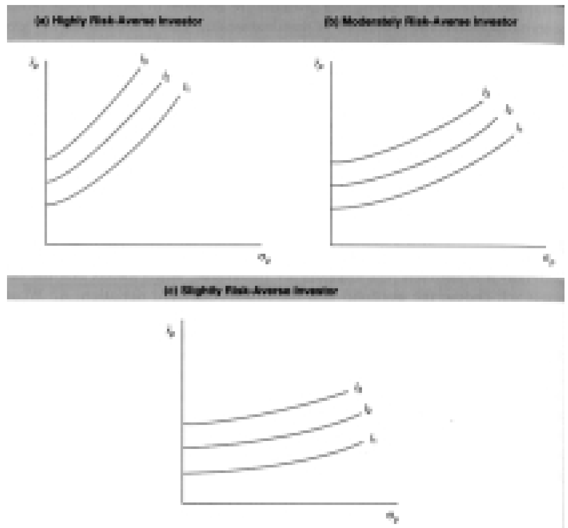

Figure 1: A high, moderately and slightly risk averse indifference curves.

vertical axis indicates reward as mesured by expected return. Using variance as relevant risk measure comes from Markowitz's paper and is always used in practice, although other possibilities have been considered (see [8].)

This definition gives us the following properties, assuming we have a 'rational investor'[1]:

- All portfolios that lie on the same indifference curve are equally desirable to the investor (even though they have different expected returns and variance.) An obvious implication is that indifference curves do not intersect.
- An investor will find any portfolio that is lying on an indifference curve that is "further northwest" to be more desirable than any portfolio lying on an indifference curve that is "not as far northwest."

But how are the indifference curves shaped? Generally it is assumed that investors are *risk averse*, which means that the investor will choose the portfolio with the smaller variance given the same return. Risk averse investors will not want to take fair gambles (where the expected payoff is zero).

These two assumptions of nonsatiation and risk aversion cause indifference curves to be positively sloped and convex.

[1] A rational investor, when given a choice between two otherwise identical portfolios, will always choose the one with the higher level of expected return. This is often called the *nonsatiation* assumption.

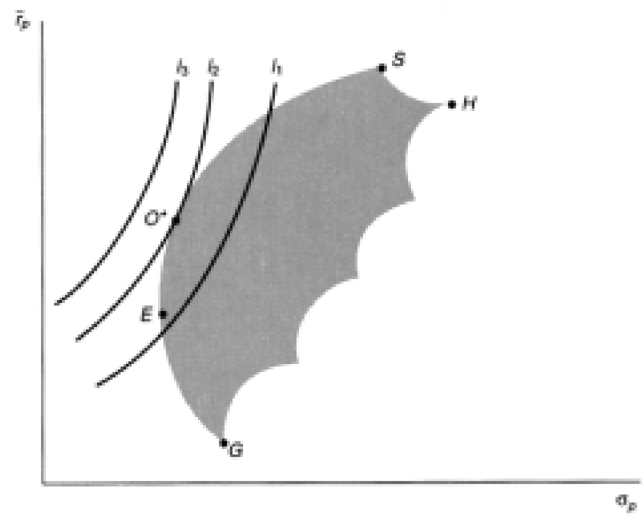

Figure 2: Graph displaying the feasible set (the dark area), the efficient set (the borderline between E and S is) and some indifference curves. The optimal portfolio is marked with O^*.

2.2 Efficient Set

Now that we know about indifference curves and risk aversion, how can we use that to select from an almost infinite number of portfolios available for investment?

The key lies in the *efficient set theorem*, which states that an investor will choose a portfolio from the set of portfolios that:

1. Offer maximum expected return for varying levels of risk, and

2. Offer minimum risk for varying levels of expected return.

We begin by constructing the *feasible set*, which represents all portfolios that could be formed from a group of N securities. The efficient set can now be located by applying the efficient set theorem to this feasible set.

This demonstrates that all the portfolios in the efficient set are located on the "northwest" boundary of the feasible set, often called the *efficient frontier*.

Selecting a portfolio is henceforth easy, by simply plotting the investor's indifference curves on the same figure as the efficient set and then proceed to choose the portfolio that is on the indifference curve that is "furthest northwest."

An important property of the efficient set is that it is concave, the proof of which is outside the scope of this paper.

2.3 CAPM

Previous sections presented a method for identifying an investor's optimal portfolio. The investor estimates the expected returns and variances for all securities under considerations and once that is done, he can simply pick the optimal portfolio for his indifference curves from the efficient frontier. Such an approach to investing is often labeled *normative economics*, since here the investors are told what they should do.

Now we enter the realm of *positive economics*, where a descriptive model of how assets are priced is presented. This model, the *Capital Asset Pricing Model* assumes that all investors use the normative approach for investing.

2.3.1 Separation Theorem

The CAPM makes the following assumptions:

- Investors evalute portfolios by looking at the variance and expected returns over a one-period horizon.
- Investors, when given a choice between two otherwise identical portfolios, will choose the one with the higher expected return.
- Investors are risk-averse.
- Individual assets are infinitely divisible.
- There is a riskfree rate at which an investor may either lend or borrow money.
- Taxes and transaction costs are negligible.

Investors are considered to be a homogeneous bunch; have the same expectations, the same one-period horizon, the same riskfree rate and that information is freely and instantly available to all investors. This is an extreme case, but it allows the focus to change from how an individual should invest to what would happen to security prices if everyone invested in a similar manner.

The first feature of the assumptions we examine is often referred to as the *separation theorem*, which states that:

Theorem 1 *The optimal combination of risky assets for an investor can be determined without any knowledge of the investor's preferences toward risk and return.*

The proof of which is pretty trivial since each person faces the same linear efficient set, where the investor will borrow or lend according to his/hers indifference curves, but the risky portion of each investor's portfolio (which we will denote by T, for *tangency portfolio*) will be the same. The linearity of the efficient set is because of the riskfree lending and borrowing introduced.

2.3.2 The Market Portfolio

From the Separation Theorem we can see that in equilibrium, every security must be part of the investor's risky portion of the portfolio. The reason is that if a security isn't in T, no one is investing in it, meaning that its prices will fall, causing the expected returns of it to rise until the resulting tangency portfolio has a nonzero proportion associated with them.

When all the price adjusting stops, the market will have been brought into equilibrium.

- Each investor will want to hold a certain positive amount of each risky security.
- The current market price of each security will be at a level where the number of shares demanded equals the number of shares outstanding.
- The riskfree rate will be at a level where the total amount of money borrowed equals the total amount of money lent.

This gives rise to the following definition of the *market portfolio:*

Definition 1 *The market portfolio is a portfolio consisting of all securities where the proportion invested in each security correspons to its relative market value. The relative market value of a security is simply equal to the aggreagte market value of the security divided by the sum of the aggregate market values of all securities.*

In equilibrium the proportions of the tangency portfolio will correspond to the proportions of the market portfolio. This tells us that the market portfolio plays a central role in the CAPM, since the efficient set consists of an investment in the market portfolio, coupled with a desired amount of either riskfree borrowing or lending.

The linear efficient set of the CAPM is known as the *Capital Market Line*, which has the following equation

$$\bar{r}_p = r_f + \left(\frac{\bar{r}_M - r_f}{\sigma_M} \right) \sigma_p \tag{1}$$

where

$$\begin{aligned}
\bar{r}_p &= \text{the expected return of an efficient portfolio,} \\
r_f &= \text{riskfree rate of return,} \\
\bar{r}_M &= \text{the expected return of the market portfolio,} \\
\sigma_M &= \text{the standard deviation of the market portfolio,} \\
\sigma_p &= \text{the standard deviation of an efficient portfolio.}
\end{aligned}$$

We now know that using the CAPM we can decide whether the market price for a stock is too high or too low by looking at the market portfolio.

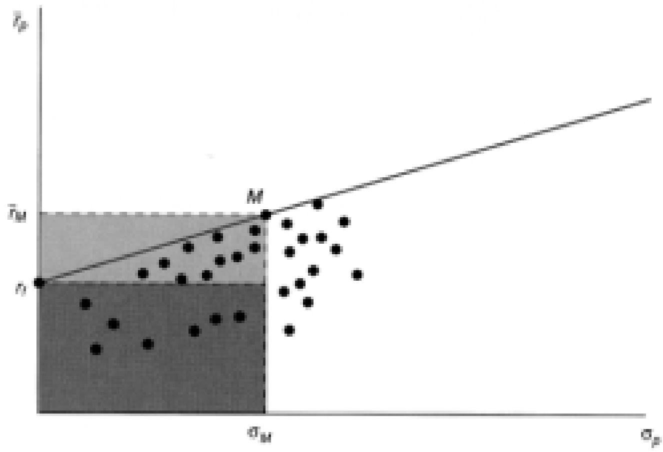

Figure 3: The Capital Market Line. M is the market portfolio and r_f represents the riskfree rate of return. All portfolios other than those employing the market portfolio and riskfree borrowing or lending would lie below the CML.

3 Portfolio Management and Information Theory

Let's denote a stock market for one investment period as $\mathbf{x} = (x_1, x_2, \ldots, x_m)^t \geq 0$, where x_i is the *price relative* for the ith stock, i.e., the ratio of closing to opening price for stock i. A portfolio $\mathbf{b} = (b_1, b_2, \ldots, b_m)^t, b_i \geq 0, \sum b_i = 1$, is the proportion of the current wealth invested in each of the m stocks. Therefore, the wealth increase over one investment period using portfolio $\mathbf{b}$ is $S = \mathbf{b}^t\mathbf{x} = \sum b_i x_i$, where $\mathbf{b}$ and $\mathbf{x}$ are considered to be column vectors.

If we consider an arbitrary sequence of stock vectors $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$, we achieve wealth

$$S_n(\mathbf{b}) = \prod_{i=1}^{n} \mathbf{b}^t \mathbf{x}_i, \tag{2}$$

with a constant rebalanced portfolio strategy $\mathbf{b}$.

The maximum wealth achievable on the given stocks is

$$S_n^* = \max_{\mathbf{b}} S_n(\mathbf{b}). \tag{3}$$

Our goal is, of course, to achieve wealth as close to S_n^* as possible.

Since we generally reinvest every day in the stock market, the accumulated wealth at the end of a n days is the product of factors, one for each day of the market. As will be shown, defining a *doubling rate* for a portfolio is a good idea.

Definition 2 *The doubling rate of a stock market portfolio* $\mathbf{b}$ *is defined as*

$$W(\mathbf{b}, F) = \int \log \mathbf{b}^t\mathbf{x}\ dF(\mathbf{x}) = E(\log \mathbf{b}^t\mathbf{x}), \tag{4}$$

where $F(\mathbf{x})$ *is the joint distribution of the vector of price relatives.*

Definition 3 *The optimal doubling rate* $W^*(F)$ *is defined as*

$$W^*(F) = \max_{\mathbf{b}} W(\mathbf{b}, F), \tag{5}$$

where the maximum is over all possible portfolios $b_i \geq 0$.

Similarly, a portfolio $\mathbf{b}^*$ *that achieves the maximum of* $W(\mathbf{b}, F)$ *is called a log-optimal portfolio.*

We can justify definition 2 by the following theorem

Theorem 2 *Let* $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$ *be i.i.d. according to* $F(\mathbf{x})$. *Then*

$$\frac{1}{n} \log S_n^* \to W^* \text{ with probability 1.} \tag{6}$$

Therefore the investor's wealth grows as 2^{nW^*} using the log-optimal portfolio.

An import property of W is that $W(\mathbf{b}, F)$ is concave in $\mathbf{b}$ and linear in F, and $W^*(F)$ is convex in F (for a proof, see [3].) That knowledge tells us that the set of log-optimal portfolios forms a convex set.

The importance of these properties will come clear in the next section.

3.1 The Log-Optimal Portfolio

The results of the Karush-Kuhn-Tucker conditions (or KKT conditions) are in the following theorem

Theorem 3 *Assume that* $f(\mathbf{x}), g_1(\mathbf{x}), g_2(\mathbf{x}), \ldots, g_m(\mathbf{x})$ *are differentiable functions (satisfying certain regularity conditions.) Then*

$$\mathbf{x}^* = (x_1^*, x_2^*, \ldots, x_n^*)$$

can be an optimal solution for the non-linear programming problem only if there exist m *numbers* $u_1, u_2, \ldots, u_m$ *such that all the following KKT conditions are satisfied:*

$$\left.\begin{array}{ll} \textit{1.} & \frac{\delta f}{\delta x_j} - \sum_{i=1}^m u_i \frac{\delta g_i}{\delta x_j} \leq 0 \\ \textit{2.} & x_j^* \left(\frac{\delta f}{\delta x_j} - \sum_{i=1}^m u_i \frac{\delta g_i}{\delta x_j} \right) = 0 \end{array}\right\} \text{at } \mathbf{x} = \mathbf{x}^*, \text{ for } j = 1, 2, \ldots, n.$$

$$\left.\begin{array}{ll} \textit{3.} & g_i(\mathbf{x}^*) - b_i \leq 0 \\ \textit{4.} & u_i[g_i(\mathbf{x}^*) - b_i] = 0 \end{array}\right\} \text{for } i = 1, 2, \ldots, m.$$

5. $x_j^* \geq 0$ *for* $j = 1, 2, \ldots, n$.

6. $u_i \geq 0$ *for* $i = 1, 2, \ldots, m$.

Which gives as a corollary, that if $f(\mathbf{x})$ is a concave function and $g_1(\mathbf{x}), g_2(\mathbf{x}), \ldots, g_m(\mathbf{x})$ are convex functions, then $\mathbf{x}^* = (x_1^*, x_2^*, \ldots, x_n^*)$ is an optimal solution iff all the conditions of the theorem are satisfied.

From this and the fact that we are trying to find an optimal solution for $\mathbf{b}$, maximizing the concave function $W(\mathbf{b}, F)$ over a convex set $\mathbf{b} \in B$, we can derive the following theorem

Theorem 4 *The log-optimal portfolio* $\mathbf{b}^*$ *for a stock market* $\mathbf{X}$ *(i.e., the portfolio that maximizes the doubling rate), satisfies the following necessary and sufficient conditions:*

$$E\left(\frac{X_i}{\mathbf{b}^{*t}\mathbf{X}}\right) \begin{array}{ll} = 1 & \text{if } b_i^* > 0, \\ \leq 1 & \text{if } b_i^* = 0. \end{array} \tag{7}$$

This tells us that the expected value of the ratio between a price relative i and the corresponding wealth relative is equal to 1 if the i component of the portfolio is non-zero, and ≤ 1 if the component is zero.

Two interesting things can be derived from this theorem. The first is that

$$E \log \frac{S}{S^*} \leq 0, \text{ for all } S \Leftrightarrow E\frac{S}{S^*} \leq 1, \text{ for all } S \tag{8}$$

where $S^* = \mathbf{b}^{*^t}\mathbf{x}$ is the random wealth resulting from the log-optimal portfolio $\mathbf{b}^*$ and S is the wealth resulting from any other portfolio $\mathbf{b}$.

We have now shown that the log-optimal portfolio, in addition to maximizing the asymptotic growth rate, also maximizes the wealth relative for one day.

Another consequence is that the expected proportion of wealth in stock i at the end of the day is the same as the proportion invested in stock i at the beginning of the day. Stated more precisely as

$$E\left(\frac{b_i^* X_i}{\mathbf{b}^{*\mathbf{t}}\mathbf{X}}\right) = b_i^* E\left(\frac{X_i}{\mathbf{b}^{*\mathbf{t}} X}\right) = b_i^* 1 = b_i^*.$$

But what if an investor were to use causal investment strategy? We can prove that with probability 1, the conditionally log-optimal investor will not do any worse than any other investor who uses a causal investment strategy. Let

$$S_n = \prod_i^n \mathbf{b}_i^t \mathbf{X}_i \tag{9}$$

be the wealth after n days for an investor who uses portfolio $\mathbf{b}_i$ on day i. Let

$$W^* = \max_{\mathbf{b}} W(\mathbf{b}, F) = \max_{\mathbf{b}} E \log \mathbf{b}^t X \tag{10}$$

be the maximal doubling rate and let $\mathbf{b}^*$ be a portfolio that achieves that rate.

From this we get

$$E \log S_n^* = nW^* \geq E \log S_n, \tag{11}$$

that is, $\mathbf{b}^*$ (satisfying equation 7) maximizes the expected log wealth and that the wealth S_n^* is equal to 2^{nW^*} to first order in the exponent, with high probability.

In fact, we can prove a much stronger result, which shows that the log-optimal portfolio will do as well or better than any other portfolio to first order in the exponent.

3.2 Side Information

What happens if we do not have the correct information to select our portfolio? This is equivalent to choosing $\mathbf{b}_g^*$, corresponding to probability density $g(\mathbf{x})$, while $\mathbf{b}_f^*$ (corresponding to probability density $f(\mathbf{x})$) is the correct one.

We can prove, given that $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$ i.i.d. $f(\mathbf{x})$, that

$$\Delta W = W(\mathbf{b}_f^*, F) - W(\mathbf{b}_g^*, F) \leq D(f||g).$$

Which tells us that the increase ΔW in doubling rate due to side information Y is bounded by the mutual information between the side information Y and the stock market $\mathbf{X}$.

$$\Delta W \leq I(\mathbf{X}; Y).$$

3.3 Stochastic Markets

The previous results were derived assuming i.i.d. markets, but can be extended to time-dependent markets.

In stochastic markets, it has been shown, with increasing levels of generality on the stochastic process, that

$$\lim_{n\to\infty} \frac{1}{n} \ln \frac{S_n}{S_n^*} \leq 0, \tag{12}$$

for every sequential portfolio. This tells us that $\mathbf{b}^*(F)$ is asymptotically optimal in this sense, and $W^*(F)$ is the highest possible exponent for the growth rate of wealth.

4 Universal Portfolios

In this section a portfolio selection procedure will be considered with the goeal of performing as well as if we knew the empirical distribution of future market performance.

The *universal adaptive portfolio strategy* is the performance weighted strategy specified by

$$\hat{\mathbf{b}}_1 = \left(\frac{1}{m}, \frac{1}{m}, \ldots, \frac{1}{m}\right), \tag{13}$$

$$\hat{\mathbf{b}}_{k+1} = \frac{\int \mathbf{b} S_k(\mathbf{b}) d\mathbf{b}}{\int S_k(\mathbf{b}) d\mathbf{b}}, \tag{14}$$

where

$$S_k(\mathbf{b}) = \prod_{i=1}^{k} \mathbf{b}^t \mathbf{x}_i$$

and the integration is over the set of $(m-1)$- dimensional portfolios

$$B = \{\mathbf{b} \in R^m : b_i \geq 0, \sum_{i=1}^{m} b_i = 1\}.$$

The wealth $\hat{S}_n$ achieved from the using the universal portfolio is

$$\hat{S}_n = \prod_{k=1}^{n} \mathbf{b}_k^t \mathbf{x}_k.$$

From this we see that the portfolio $\hat{\mathbf{b}}_1$ is uniform over the stocks, and the portfolio $\hat{\mathbf{b}}_k$ at time k is the performance weighted average of all portfolios $\mathbf{b} \in B$.

As in the previous section we have

$$S_n^* = \max_b s_n(\mathbf{b}) = \max_b \prod_{i=1}^{n} \mathbf{b}^t \mathbf{x}_i = e^{nW^*(F_n)}, \tag{15}$$

where F_n denotes the empirical distribution of $\mathbf{x}_1, \ldots, \mathbf{x}_n$, i.e. it places mass $\frac{1}{n}$ at each $\mathbf{x}_i$.

We can easily prove that S_n^* exceeds the maximum of the component stocks, the arithmetic mean, the geometric mean and that $S_n^*(\mathbf{x}_1, \ldots, \mathbf{x}_n)$ is invariant under permutations of the sequence $\mathbf{x}_1, \ldots, \mathbf{x}_n$.

- The target wealth exceeds the wealth from best stock:

$$S_n^* \geq \max_{j=1,\ldots,m} S_n(\mathbf{e}_j). \tag{16}$$

- The target wealth exceeds the Dow Jones (or the arithmetic mean):

$$S_n^* \geq \sum_{j=1}^{m} \alpha_j S_n(\mathbf{e}_j), \tag{17}$$

  where $\alpha_j \geq 0, \sum \alpha_j = 1$.

  This assumes buy-and-hold strategies $\mathbf{b} = \mathbf{e}_j = (0, \ldots, 0, 1, 0, \ldots, 0)$, where e_j is the j-th basis vector.

Since the wealth $\hat{S}_n$, resulting from the universal portfolio, is the average of $S_n(\mathbf{b})$, or

$$\hat{S}_n = \prod_{k=1}^{n} \hat{\mathbf{b}}_k^t \mathbf{x}_k = \frac{\int S_n(\mathbf{b}) d\mathbf{b}}{\int d\mathbf{b}}, \tag{18}$$

where

$$S_n(\mathbf{b}) = \prod_{i=1}^{n} \mathbf{b}^t \mathbf{x}_i,$$

we can show that for the universal portfolio

$$\hat{S}_n \geq \left(\prod_{j_1}^{m} S_n(\mathbf{e}_j) \right)^{1/m}, \tag{19}$$

or, the wealth from the universal portfolio also exceeds *value line index.*

We can also prove that $\hat{S}_n$ is invariant under permutations of the sequence $\mathbf{x}_1, \ldots, \mathbf{x}_n$. This invariance entails that a stock market crash will have no worse consequences for wealth $\hat{S}_n$ than if the bad days of that time had been sprinkled out among the good.

The important question remains, how does $\hat{S}_n / S_n^*$ behave? For a portfolio of two stocks, we consider the arbitrary stock vector sequence

$$\mathbf{x}_i = (x_{i1}, x_{i2}) \in \mathbf{R}_+^2, i = 1, 2, \ldots. \tag{20}$$

The portfolio choice can be transformed into a choice of one variable, namely

$$\mathbf{b} = (b, 1-b),\ 0 \leq b \leq 1, \tag{21}$$

and similarly $S_n(\mathbf{b})$ becomes

$$S_n(b) = \prod_{i=1}^{n} (b x_{i1} + (1-b) x_{i2}),\ 0 \leq b \leq 1. \tag{22}$$

Let's define

$$J_n^* = -W^{''}(b_n^*) = \int \frac{(x_{i1} - x_{i2})^2}{(\mathbf{b}_n^{*^t} \mathbf{x})^2} dF_n(\mathbf{x}). \tag{23}$$

J_n is generally known as the curvature or volatility index of a portfolio.

It can be shown that for any $\mathbf{x}_1, \mathbf{x}_2, \ldots \in \mathbf{R}_+^2$ and for any subsequence of times $n_1, n_2, \ldots$ such that the doubling rate $W_n(b)$ satisfies the condition

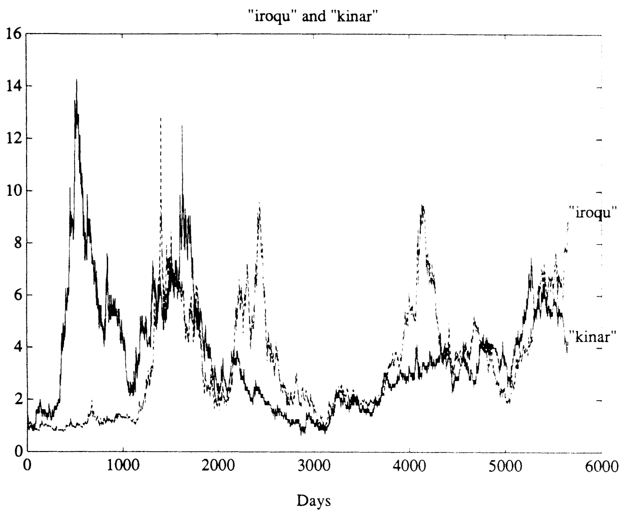

Figure 4: Performance of Iroquois Brands Ltd. and Kin Ark Corp. stock during the time period from 1963 to 1985.

$$W_n''(b_n^*) \to W''(b^*), \tag{24}$$

and where $W(b)$ achieves its maximum at $b^* \in (0,1)$ the following holds true

$$\frac{\hat{S}_n}{S_n^*} \sim \sqrt{\frac{2\pi}{nJ_n^*}}. \tag{25}$$

This means that the universal wealth is within a factor $C/\sqrt{n}$ of the (presumably) exponentially large S_n^*. In fact, it can be further shown that every additional stock in the universal portfolio costs an additional factor of $1/\sqrt{n}$.

4.1 Real world example

Let's now consider how this portfolio algorithm would perform on two real stocks. We'll consider a 22-year period (ending in 1985) of the stock of Iroquois Brands Ltd. and Kin Ark Corp., which were highly volatile during that time period as can be seen in figure 4.

If an investor would have had access to this information in 1963 he could have earned approx. 791% profit by buying and holding the best stock (Iroquois) which has an ending rate of 8.915 (see figure 4).

If we look at the performance of some constant rebalanced portfolios for that period (see table 1), we notice that the best rebalanced portfolio is $\mathbf{b}^* =$

b	$S_n(b)$	b	$S_n(b)$
1.00	8.9151	0.45	68.0915
0.95	13.7712	0.40	60.7981
0.90	20.2276	0.35	51.6645
0.85	28.2560	0.30	41.7831
0.80	37.5429	0.25	32.1593
0.75	47.4513	0.20	23.5559
0.70	57.0581	0.15	16.4196
0.65	65.2793	0.10	10.8910
0.60	71.0652	0.05	6.8737
0.55	73.6190	0.00	4.1276
0.50	72.5766		

Table 1: Constant rebalanced portfolio performance of the The Iroquois Brands Ltd. vs. Kin Ark Corp. for different portfolio strategies.

$(.55, .45)$, which gives us wealth increase of $S_n^* = 73.619$. This is the target wealth, which we strive to get as close to as possible.

To use the universal portfolio algorithm, described in the previous section, we must for quantize all integrals giving us the following equations:

$$S_n^* = \max_{i=0,1,\ldots,20} S_n(i/20), \tag{26}$$

$$\hat{b}_{k+1} = \frac{\sum_{i=0}^{20} \frac{i}{20} S_k(\frac{i}{20})}{\sum_{i=0}^{20} S_k(\frac{i}{20})}, \tag{27}$$

and wealth

$$\hat{S}_n = \prod_{k=1}^{n} \hat{b}_k x_k. \tag{28}$$

It can be verified that $\hat{S}_n$ can be expressed in the equivalent form

$$\hat{S}_n = \frac{1}{21} \sum_{i=0}^{20} S_n\left(\frac{i}{20}\right). \tag{29}$$

Using this we can get a universal wealth of $\hat{S}_n = 38.6727$ as can be seen in figure 5. Even though the universal portfolio gives less wealth then S_n^* it still gives much greater wealth than what an investor could get, when given information n days into the future.

Although these results are encouraging, the universal portfolio barely outperforms stocks with a lockstep performance, e.g. the stocks in Coca Cola and IBM.

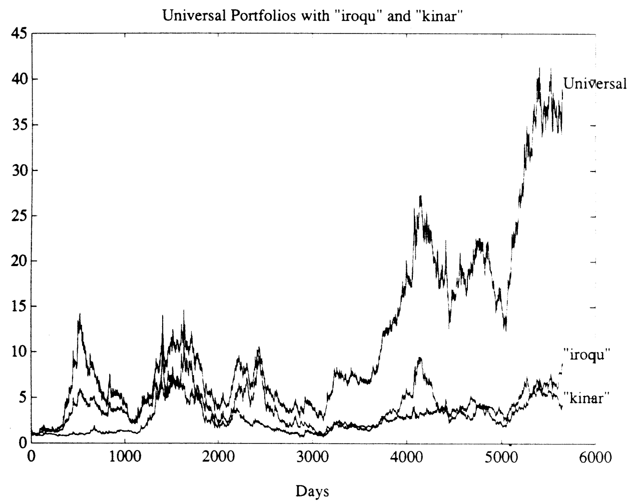

Figure 5: Performance of the universal portfolio compared to the performance of stocks in The Iroquois Brand Ltd. and Kin Ark Corp.

5 Summary

My goal for this project was to familiarize myself with the intricacies of Portfolio Management and in particular how Information Theory could be utilized in this regard. This required extensive reading, both in general investment theory and information theory. I hope that most readers will gain a greater insight into Portfolio Managment by reading this paper.

All that remains now is to make a fortune in the stock market using this new found knowledge. The only drawback is the the universal portfolio algorithm does not take into account transaction fees, which are the bane of many an investor.

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