

# Jaap's Puzzle Page

## Babylon Tower



The Babylon Tower, sometimes called the Ivory Tower, is a sliding piece puzzle which consists of several discs stacked up into a tower, and they can rotate about a central axle. There are 6 such discs. Along the side of the tower are six columns of small balls in a pieces, in six colours. The colours in each column range from bright in the lower disc to pale in the top one. The bottom disc has a spring allowing one of two opposite balls to be pushed in, creating a gap. The balls can then slide up or down their groove from one disc to another because of the gap. By rotating the discs, the balls and the gap are moved around to the other columns.

This puzzle was designed by Endre Pap, who also designed the [Hungarian Rings](#). The patent, [DE 3,104,021](#) from 2 December 1982, only lists the assignee Arxon Spiel + Freizeit.

Other related puzzles are the [Whip-It Tower](#), and [Missing Link](#).

### The number of positions:

There are 36 balls and a gap which gives a maximum of  $37!$  positions. However the colours are equivalent, meaning that it does not matter which colour goes in which column. This gives  $37! / 6! = 19,116,323,737,814,368,119,883,304,974,417,920,000,000$  or  $1.9 \cdot 10^{40}$  positions.

### Solution 1:

It can be solved in layers, disc by disc from the bottom up starting with the brightest colours. Start by pushing in one of the balls to create a gap.

#### Phase 1: How to solve a disc.

- a. Find a ball that belongs in the disc but which is not in the correct position.
- b. If the ball is in the disc already, just in the wrong place then move it to the layer above:

1. Shift balls down to get the gap in one of the discs above the ball.
  2. Rotate discs to place the gap above the ball in the same column.
  3. Shift the ball up.
  4. Rotate the ball to a different column and drop another ball in the gap.
- c. If the ball does not lie one layer immediately above the disc being solved then:
1. Rotate a disc to place the gap in a different column to the ball.
  2. Shift the column up/down to get the gap in the disc below the ball.
  3. Rotate discs to place the gap beneath the ball.
  4. Shift the ball down.
- Repeat until the piece lies in the layer immediately above the disc being solved.
- d. Place the piece correctly as follows:
1. Rotate discs to place the piece in a different column than its destination, and the gap in the destination column.
  2. Shift the column up/down to get the gap in the disc below the piece.
  3. Rotate discs to place the gap beneath the piece.
  4. Shift the piece down.
- Note the similarity to step b.
- e. Repeat a-d above until the whole disc is solved.

The method above can solve any of the layers of the tower, except for the top one. There are a few minor differences when solving the first layer because of the spring mechanism that creates the gap. First push in one ball to create the gap. Solve the ball on the other side. Then pull out the first ball and push in the solved one. Now continue the method as above.

**Phase 2:** Solve all but top disc.

Repeat phase 1 with the remaining discs until everything except the last disc is solved.

**Phase 3:** Solve top disc.

- a. If the permutation of the balls and the gap (where the gap belongs in the column of the pushed-in ball) is odd, then rotate the disc one step, i.e. 1/6 of a turn.
- b. Rotate the top disc an even number of steps (i.e. 1/3 turn) to put as many pieces correct as possible.
- c. To solve the remaining balls, the following sequences will come in handy.

Let a and A denote the top balls of one column, b and B those of another column etc., and let Z be top ball of the column with the gap. The following diagrams show how certain positions can be solved:

B A	BaA	Ba	Bab	ab	Aab	A b	ABb	AB		
abZ	bZ	AbZ	A Z	ABZ	BZ	aBZ	a Z	abZ		
BCA	BCAa	CAa	cCAa	c Aa	cBAa	cB a	cBCa	BCa	ABCa	ABC
abcZ	bcZ	BbcZ	Bb Z	BbcZ	bcZ	AbcZ	Ab Z	AbcZ	bcZ	abcZ

Both of these cycle three things in the top disc, the first cycles two balls and the gap, the second three balls. Note that the relevant columns need not be adjacent nor in the order as shown here. Simply mentally name the columns a, b, Z (and c) and move the balls between these columns as shown above wherever they may physically be on the puzzle. From step a, the balls have an even permutation, so these 3-cycles suffice to solve it. If you end up with exactly two balls swapped (or only a ball and the gap swapped) then it was an odd permutation so go back to step a to rectify it.

**Solution 2:**

It can also be solved in columns.

**Phase 1:** Place each colour into a column.

This phase is virtually the same as solving the [Whippit](#) puzzle.

- a. Decide which colours the columns will have. To speed things up, you could rotate some discs to partly solve some columns already.
- b. Push in a ball to create a gap.
- c. Look at the column containing the gap, and determine which colour the column has.
- d. Find a piece of the required colour in a different column. If there are none, and the columns are not yet one colour each then find any piece on the puzzle that is incorrect.
- e. Slide pieces in the column up or down until the gap is in a layer next to the layer that contains the piece.
- f. Rotate the layer with the gap until it lies above or below the piece.
- g. Slide the piece up or down into the gap.
- h. Rotate the layer back to its original position, which brings the piece into the column as required.
- i. Repeat steps b-g until the puzzle is solved. At some point you should also let out the pushed in ball, and push in the ball on the other side. Do this any time that the bottom two balls of the column on the other side have been solved.

Note that in practice it is quicker to move not just the one layer, but all the layers above/below it as well.

**Phase 2:** Order each colour, except for top two layers.

- a. There now follows a method of moving a ball up two layers, passing over two balls which drop down one layer. This is a 3-cycle.  
First create a gap in any other column.
- b. Bring the gap to the layer above the ball to move.
- c. Use the method of phase 1 to bring the ball to the position of the gap. (Rotate the gap above the ball, move ball up, rotate disc back again.)
- d. Bring the gap to the layer above the ball to move.
- e. Again insert the ball into the position where the gap is.
- f. Use steps b-e to place the correct 4 balls in the bottom 4 positions of the column. Note that the top two balls may now either be correct, or they need to be swapped.
- g. Now get rid of the gap by releasing the pushed in ball.
- h. Solve each column in the same way.

**Phase 3:** Swap the top two balls of any columns that need it.

- a. If there is an odd number of columns that need their top balls swapped, then turn the top layer 60 degrees (i.e. one step) and go back to phase 1. It will not take long to fix this problem since the bottom 4 layers need not be disturbed.
- b. If there are two or more columns that need swaps, then choose any two of them, and create a gap in a third column.
- c. Do the sequence shown in the following diagrams.

ab	abA	a A	aBA	aB	aBb	Bb	ABb	AB
ABZ	BZ	bBZ	b Z	bAZ	AZ	aAZ	a Z	abZ

Note that the relevant columns need not be adjacent nor in the order shown in these diagrams. Simply mentally name the columns a, b, and Z and move the balls between these columns as shown above wherever they may physically be on the puzzle.

- d. Repeat steps b-c until the puzzle is solved.

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