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Algebra
Applied Mathematics
Calculus and Analysis
Discrete Mathematics
Foundations of Mathematics
Geometry
History and Terminology
Number Theory
Probability and Statistics
Recreational Mathematics
Topology

Alphabetical Index
Interactive Entries
Random Entry
New in *MathWorld*

MathWorld Classroom

About *MathWorld*
Contribute to *MathWorld*
Send a Message to the Team

MathWorld Book

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Discrete Mathematics > Coding Theory >
Interactive Entries > Interactive Demonstrations >

Gray Code

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A Gray code is an encoding of numbers so that adjacent numbers have a single digit differing by 1. The term Gray code is often used to refer to a "reflected" code, or more specifically still, the binary reflected Gray code.

To convert a binary number $d_1 d_2 \cdots d_{n-1} d_n$ to its corresponding binary reflected Gray code, start at the right with the digit d_n (the nth, or last, digit). If the d_{n-1} is 1, replace d_n by $1 - d_n$; otherwise, leave it unchanged. Then proceed to d_{n-1}. Continue up to the first digit d_1, which is kept the same since d_0 is assumed to be a 0. The resulting number $g_1 g_2 \cdots g_{n-1} g_n$ is the reflected binary Gray code.

To convert a binary reflected Gray code $g_1 g_2 \cdots g_{n-1} g_n$ to a binary number, start again with the nth digit, and compute

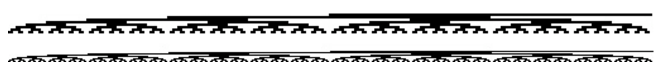
$$\Sigma_n \equiv \sum_{i=1}^{n-1} g_i \pmod 2.$$

If Σ_n is 1, replace g_n by $1 - g_n$; otherwise, leave it the unchanged. Next compute

$$\Sigma_{n-1} \equiv \sum_{i=1}^{n-2} g_i \pmod 2,$$

and so on. The resulting number $d_1 d_2 \cdots d_{n-1} d_n$ is the binary number corresponding to the initial binary reflected Gray code.

The code is called reflected because it can be generated in the following manner. Take the Gray code 0, 1. Write it forwards, then backwards: 0, 1, 1, 0. Then prepend 0s to the first half and 1s to the second half: 00, 01, 11, 10. Continuing, write 00, 01, 11, 10, 10, 11, 01, 00 to obtain: 000, 001, 011, 010, 110, 111, 101, 100, ... (Sloane's A014550). Each iteration therefore doubles the number of codes.



The plots above show the binary representation of the first 255 (top figure) and first 511 (bottom figure) Gray codes. The Gray codes corresponding to the first few nonnegative integers are given in the following table.

0	0	20	11110	40	111100
1	1	21	11111	41	111101
2	11	22	11101	42	111111
3	10	23	11100	43	111110
4	110	24	10100	44	111010
5	111	25	10101	45	111011
6	101	26	10111	46	111001
7	100	27	10110	47	111000
8	1100	28	10010	48	101000
9	1101	29	10011	49	101001
10	1111	30	10001	50	101011
11	1110	31	10000	51	101010
12	1010	32	110000	52	101110
13	1011	33	110001	53	101111
14	1001	34	110011	54	101101
15	1000	35	110010	55	101100
16	11000	36	110110	56	100100
17	11001	37	110111	57	100101

Interactive Demonstrations

Gray Indexed Minimum Change Permutation

Mixed Radix Number Representations

Elementary Cellular Automaton Rules by Gray Code

Mixed Base Gray Codes

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18	11011	38	110101	58	100111
19	11010	39	110100	59	100110

The binary reflected Gray code is closely related to the solutions of the towers of Hanoi and baguenaudier, as well as to Hamiltonian circuits of hypercube graphs (including direction reversals; Skiena 1990, p. 149).

SEE ALSO: Baguenaudier, Binary, Hilbert Curve, Ryser Formula, Thue-Morse Sequence, Towers of Hanoi

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