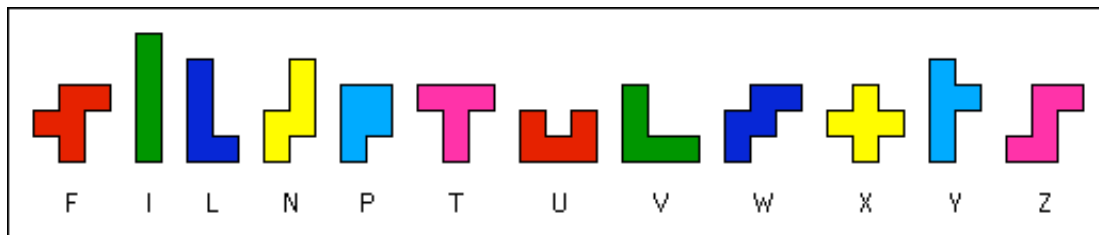


Welcome to Gerard's Pentomino Page

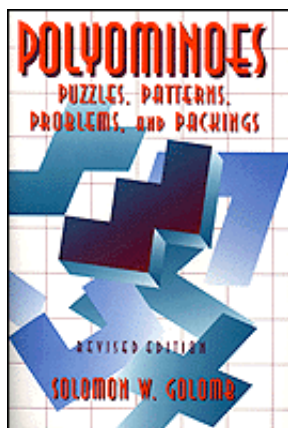


Introduction

The basic set of pentominoes consists of twelve different pieces (excluding the ones that are identical by rotating and flipping).



To make it easier to talk about the pieces, they are given names, as indicated. These names were first proposed by Solomon W. Golomb in his excellent book "Polyominoes".



This book discusses polyominoes in general, and has a large section about pentominoes. Solomon Golomb can be considered the "inventor" or rather the "discoverer" of the concept of polyominoes, back in 1953.

In the book, numerous problems (puzzles) are described, some of which are not yet solved. Furthermore, it contains a number of interesting and sometimes amazing facts about pentominoes. One of them is the stunning number of solutions for the problem of fitting the 12 pentominoes into a rectangle. As each piece covers an area of 5 "units", all pieces together cover 60 units. The different rectangles that can be made out of 60 squares measure 1x60, 2x30, 3x20, 4x15, 5x12 and 6x10. Of these, the formats 1x60 and 2x30 can not be covered with the 12 pentominoes, for obvious reasons (consider the 'X' piece, for example, which is 3 units wide in every direction). Surprisingly, the 3 x 20 format **can** be solved, although the solution is extremely difficult to find; only 2 solutions exist. When Golomb's book was first published in 1965, the number of solutions of the four possible rectangular puzzles had all been determined by a computer program. Nowadays, everybody with a personal computer and enough programming skills can verify the results, and produce a listing of all possible solutions. This is exactly what I did in 1982, using my first computer, an Apple II (1MHz 6502 processor, 64K RAM). I wrote the program in Pascal, and was able to find all solutions in about 3 weeks (!) time. In 1988, I created a universal program, written in ANSI C, fit to solve ANY polyomino puzzle (any number of pieces, any shape of pieces, any shape of the solution). This highly optimized "solution engine" can be compiled and run on any computer that has an ANSI C compiler. On my current computer, a Power Macintosh G3 (266 MHz, 128Mb RAM), the same C program generates all solutions in just over 1 minute. Now that is progress! Anyway, the number of solutions for each of the rectangles is:

- 3 x 20.....2
- 4 x 15.....368

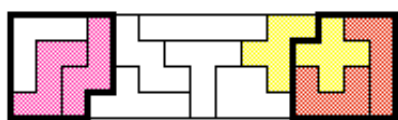
- 5 x 12.....1010
- 6 x 10.....2339

Of course, these numbers do not include the solutions that are trivial transformations (flip / rotate) of another solution. My own standard way to avoid trivial solutions, is to normalize the position of the 'F' piece: it must always be oriented in the position shown in the first picture, or rotated 90 degrees clockwise.

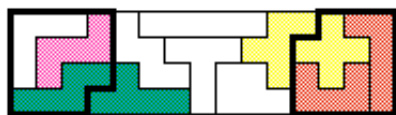
I also have a Java version of the solution program, check out the [Polyomino puzzle solver page](#)

Symmetrical combinations

In many solutions, you often find "symmetrical combinations". This is a combination of exactly two pieces, that together can be rotated or flipped to cover the exact same shape in a different way. To give an example of a 4 x 15 solution that contains an extreme amount of symmetries:



The symmetrical combinations are indicated by colors. When the "N - Z" combination is flipped, a new symmetrical combination (symcom) occurs, namely the N - Y combination:



Apart from the symcoms, this solution also contains two areas with identical shapes, which I indicated with a thick border. Obviously, these two groups can be swapped to form even more solutions:



Now suddenly the special "N - F" combination appears. Without being a symmetrical combination, these two pieces can still fill the same area in more than one way, as indicated. As far as I know, this is only other pair of pieces that has this property, namely the L and the P. All in all, using symmetrical combinations, swapping identical subparts, and the N - F combination, in this example we have a group of 18 related solutions.

Apparently, symmetrical combinations are important to recognize additional solutions. Therefore, the following problems are interesting (and NOT completely solved yet, as far as I know):

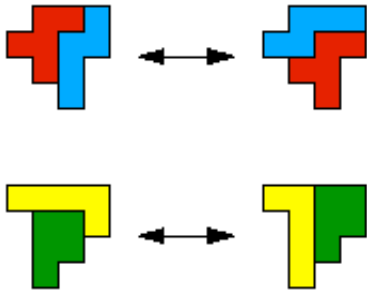
1. Find all possible symmetrical combinations of two pentominoes.
2. For each of the symmetrical combinations found in (1), find a rectangular solution that contains it.

As a pure symcom exercise: fit all 12 pentominoes together, forming any shape, so that each pentomino touches at least one other pentomino, and each pentomino forms a symmetrical combination with every pentomino it touches. 'Touching' means: to have at least one unit length in common.

A long time ago, I tried to find the solutions for (1) manually, and I am rather convinced I found them all

(but I am not sure). For problem (3) I found one solution, and I doubt if there are any more.

The only pairs of pieces that can fill the same area in two different ways are N-F and L-P:



Other references

[This page of Kadon Enterprises](#) has an overview of many interesting Internet sites about polyomino puzzles.

[Gerard's home page](#)