Pentomino

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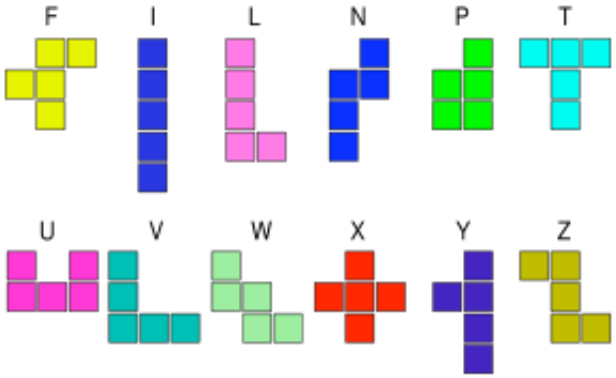


A **pentomino** is a polyomino composed of five (Greek *πέντε* / *pente*) congruent squares, connected orthogonally.

There are twelve different pentominoes, and they are named after the letters of the Latin alphabet that they resemble. Ordinarily, the reflection symmetry and rotation symmetry of a pentomino does not count as a different pentomino.

F, L, N, P, Y, and Z pentominoes are chiral in two dimensions; adding their reflections (F', J, N', Q, Y', S) brings the number of "one-sided" pentominoes to 18. The others, lettered I, T, U, V, W, and X, are equivalent to some rotation of their mirror images. This matters in some computer games, where mirror image moves are not allowed, such as Tetris-clones and Rampart.

Each of the twelve pentominoes can be tiled to fill the plane. In addition, each chiral pentomino can be tiled without using its reflection.

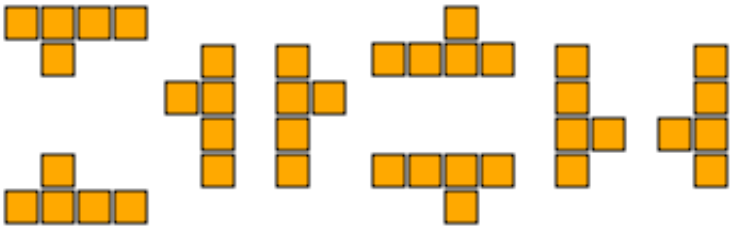
John Horton Conway proposed an alternate labeling scheme -- he uses O instead of I, Q instead of L, R instead of F, and S instead of N. The resemblance to the letters is a bit more strained, but this scheme has the advantage that it uses 12 consecutive letters of the alphabet. In reference to Conway's Game of Life, this scheme is used, so it talks about the R-pentomino instead of the F-pentomino.

Considering rotations of multiples of 90 degrees only, there are the following symmetry categories:

- L, N, P, F and Y can be oriented in 8 ways: 4 by rotation, and 4 more for the mirror image.
- Z can be oriented in 4 ways: 2 by rotation, and 2 more for the mirror image.
- T, V, U and W can be oriented in 4 ways by rotation.
- I can be oriented in 2 ways by rotation.
- X can be oriented in only one way.

For 2D figures in general there is one more category: being orientable in 2 ways, which are each other's mirror image, for example a swastika. There is no pentomino in this category (this type of symmetry requires at least an octomino).

For example, the eight possible orientations of the Y pentomino are as follows:



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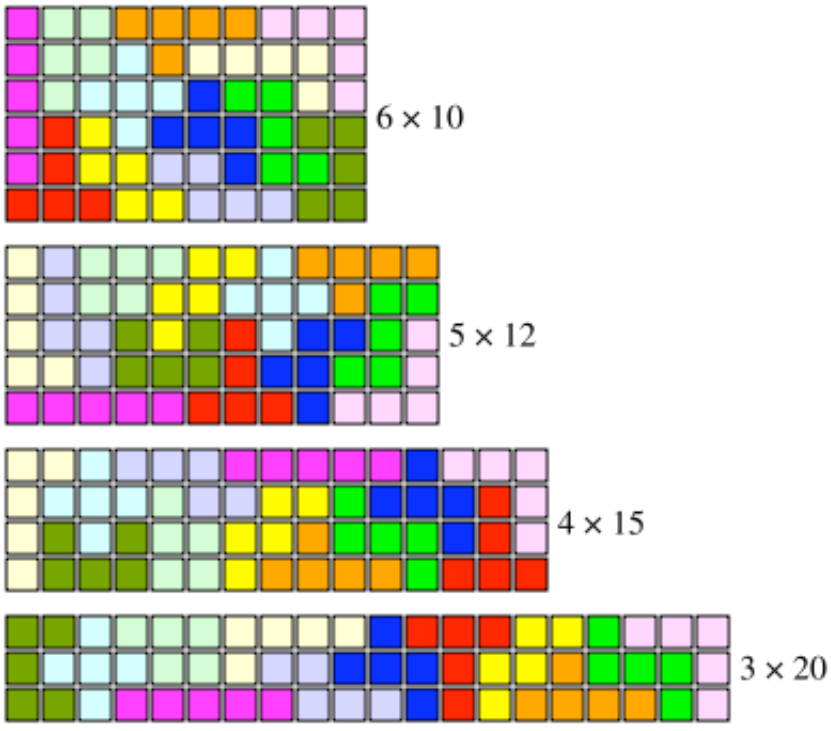
Tiling rectangles

A standard **pentomino puzzle** is to tile a rectangular box with the pentominoes, i.e. cover it without overlap and without gaps. Each of the 12 pentominoes has an area of 5 unit squares, so the box must have an area of 60 units. Possible sizes are 6×10, 5×12, 4×15 and 3×20. The avid puzzler can probably solve these problems by hand within a few hours. A more challenging task, typically requiring a computer search, is to count the total number of solutions in each case.

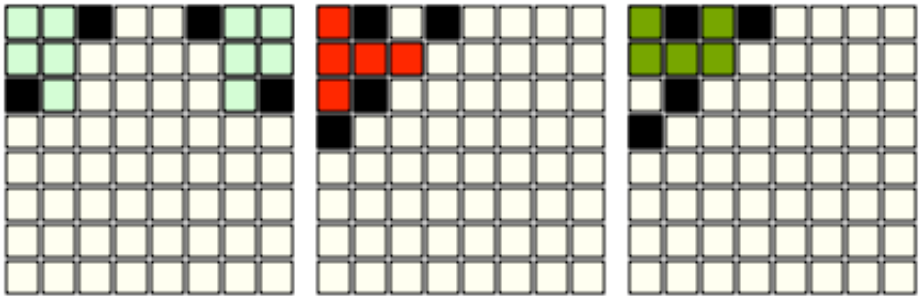


The 6×10 case was first solved in 1960 by C. B. Haselgrove and Jenifer Haselgrove .[1] There are exactly 2339 solutions, excluding trivial variations obtained by rotation and reflection of the whole rectangle, but including rotation and reflection of a subset of pentominoes (sometimes this is possible and provides in a simple way an additional solution; e.g., with the 3×20 solution shown, the other one is obtained by rotating a set of seven pentominoes, or put differently, by rotating the four

leftmost and the rightmost to the other side).

The 5×12 box has 1010 solutions, the 4×15 box has 368 solutions, and the 3×20 box has just 2 solutions.

A somewhat easier (more symmetrical) puzzle, the 8×8 rectangle with a 2×2 hole in the center, was solved by Dana Scott as far back as 1958[2]. There are 65 solutions. Scott's algorithm was one of the first applications of a backtracking computer program ever. Variations of this puzzle allow the four holes to be placed in any position. One of the external links uses this rule. Most such patterns are solvable, with the exceptions of placing each pair of holes near two corners of the board in such a way that both corners could only be fitted by a P-pentomino, or forcing a T-pentomino or U-pentomino in a corner such that another hole is created.



Efficient algorithms have been described to solve such problems, for instance by Donald Knuth[3]. Running on modern hardware, these pentomino puzzles can now be solved in mere seconds.

Filling boxes

A **pentacube** is a polycube of five cubes. Twelve of the 29 pentacubes correspond to the twelve pentominoes extruded to a depth of one square. A **pentacube puzzle** or 3D **pentomino puzzle**, amounts to filling a 3-dimensional box with these 1-layer pentacubes, i.e. cover it without overlap and without gaps. Each of the 12 pentacubes consists of 5 unit cubes, and are like 2D pentominoes but with unit thickness. Clearly the box must have a volume of 60 units. Possible sizes are 2×3×10, 2×5×6 and 3×4×5. Following are several solutions.

```
2 x 3 x 10 box

P P F N N W T U X U   V V V Z N N N U U U
P P F F W W T X X X   V Z Z Z Y I I I I I
P F F W W T T T X L   V Z Y Y Y Y L L L L

2 x 5 x 6 box

P P P N N N   P P L L L L
Y W N N X U   F F L Z Z U
Y W W X X X   V F F Z T U
Y Y W W X U   V F Z Z T U
Y I I I I I   V V V T T T

3 x 4 x 5 box

F F V V V   X F F P T   U F U P P   U U U P P
X N N N V   X L T T T   X L L L L   I I I I I
N N Z Z V   X W W Z T   W W Y Z Z   W Y Y Y Y
```

Alternatively one could also consider combinations of five cubes which are themselves 3D, i.e., are not part of one layer of cubes. However, in addition to the 12 extruded pentominoes, 6 sets of chiral pairs and 5 pieces make total 29 pieces, resulting 145 cubes, which will not make 3D box. Furthermore, it will be a computer puzzle.

Trivia

Pentominoes are a key part of the 2003 children's novel *Chasing Vermeer* by Blue Balliett and its 2005 sequel, The Wright 3. The plot revolves around pentominoes and there is a puzzle for readers to solve throughout the book that is based on pentonimoes.

Pentominoes are prominently featured in a subplot of the novel *Imperial Earth* by Arthur C. Clarke.

"Pentominoes" was registered as a trademark by Solomon W. Golomb (#1008964 USPTO 1975 April 15), but this trademark is no longer in effect as of 1982.

Other patterns that 9 of the 12 Pentomino pieces will solve, are the shapes of each of the 12 pieces enlarged x3. The German Wikipedia article mentions this too, giving examples in images.

A puzzle similar to Pentomino, also with 12 pieces, but each made of 6 equilateral triangles, exists under the name Hexiamond. The pieces must cover a 6x6 parallelogram divided in 72 equilateral triangles.

Gabriel Industries of Hagerstown, Maryland US sold a pentomino set as a puzzle under the trade name *Hexed*, along with a line of other geometry puzzles, including a tangram set called *Pythagoras* and *Crazy Quilt* (which was based on irregular shapes).

Board game

There is a board game of skill based entirely on pentominoes, called **pentominoes**.

The game is played on an 8×8 grid by two or three players. Players take turns in placing pentominoes on the board so that they do not overlap with existing tiles and no tile is used more than once. The objective is to be the last player to place a tile on the board.

The two-player version has been weakly solved; it is a first-player win.

Pentominoes, and similar shapes, are also the basis of a number of other tiling games, patterns and puzzles. For example, a French board game called Blokus is played with 4 opposing color sets of polyominoes. In Blokus, each color begins with every pentomino (12), as well as every tetromino (5), every tromino (2), every domino (1) , and every monomino (1). Like the game Pentominoes, the goal is to use all of your tiles, and a bonus is given if the monomino is saved for the very last move. The player with the fewest blocks remaining wins.

Parker Brothers released a multi-player pentomino board game called Universe in 1966. Its theme is based on

an outtake from the movie 2001: A Space Odyssey in which the astronaut (seen playing chess in the final version) is playing a two-player pentomino game against a computer. The front of the board game box features scenes from the movie as well as a caption describing it as the "game of the future". The game comes with 4 sets of pentominoes (in red, yellow, blue, and white). The board has two playable areas: a base 10x10 area for two players with an additional 25 squares (two more rows of 10 and one offset row of 5) on each side for more than two players.

Video games

- *Tetris* was inspired by pentomino puzzles, although it uses four-block tetrominoes.
- *Daedalian Opus* uses pentomino puzzles throughout the game.
- *Yohoho! Puzzle Pirates* carpentry minigame is based on pentomino puzzles.

See also

- Tiling puzzle
- Sudoku variants
- Other block puzzles (http://www.livecube.com)

References

1. **^** C. B. Haselgrove; Jenifer Haselgrove (October 1960). "A Computer Program for Pentominoes". *Eureka* **23**: 16–18.
2. **^** Dana S. Scott (1958). "Programming a combinatorial puzzle". Technical Report No. 1, Department of Electrical Engineering, Princeton University.
3. **^** Donald E. Knuth. "Dancing links" (http://www-cs-faculty.stanford.edu/~knuth/papers/dancing-color.ps.gz) (Postscript, 1.6 megabytes). Includes a summary of Scott's and Fletcher's articles.

- Pentominoes as the origin of Tetris (http://www.tetris-today.com/story/original-tetris1.shtml)
- Chasing Vermeer (http://www.scholastic.com/chasingvermeer) , with information about the book Chasing Vermeer and a click-and-drag pentomino board.
- Pentominoes: A First Player Win (http://www.msri.org/publications/books/Book29/files/orman.pdf) , by HILARIE K. ORMAN

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