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Cuboctahedron

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A **cuboctahedron** is a polyhedron with eight triangular faces and six square faces. A cuboctahedron has 12 identical vertices, with two triangles and two squares meeting at each, and 24 identical edges, each separating a triangle from a square. As such it is a quasiregular polyhedron, i.e. an Archimedean solid, being vertex-transitive and edge-transitive.

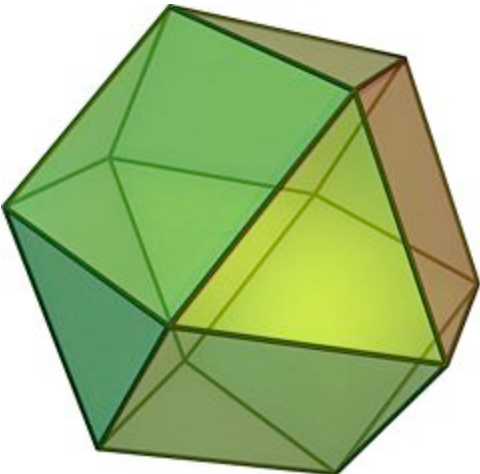
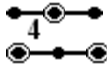
Its dual polyhedron is the rhombic dodecahedron.

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Other Names

- *Heptaparallelohedron* (Buckminster Fuller) [1] (<http://dymaxion.org/dymaxion.shtml>)
 - Fuller applied the name "Dymaxion"[2] (<http://dymaxion.org/dymaxion.shtml>) and *Vector Equilibrium* [3] (<http://www.angelfire.com/mt/marksomers/92.html>) to this shape, used in an early version of the Dymaxion map.
- *Rectified cube* or *rectified octahedron* (Norman Johnson)
 - Also *cantellated tetrahedron* by a lower symmetry.
- *Triangular gyrobicupola* (analog to Triangular orthobicupola)

Cuboctahedron	
 <p>(Click here for rotating model)</p>	
Type	Archimedean solid
Elements	$F = 14, E = 24, V = 12$ ($\chi = 2$)
Faces by sides	$8\{3\}+6\{4\}$
Schläfli symbol	$t_1\{4,3\}$ $t_{0,2}\{3,3\}$
Wythoff symbol	$2\ 1\ 3\ 4$ $3\ 3\ 1\ 2$
Coxeter-Dynkin	
Symmetry	O_h and T_h
References	U_{07}, C_{19}, W_{11}
Properties	Semiregular convex quasiregular

Area and volume

The area A and the volume V of the cuboctahedron of edge length a are:

$$A = (6 + 4\sqrt{3})a^2 \approx 12.92820323a^2$$

$$V = \frac{5}{3}\sqrt{2}a^3 \approx 2.3570226a^3.$$

Geometric relations

A cuboctahedron can be obtained by taking an appropriate cross section of a four-dimensional cross-polytope.

A cuboctahedron has octahedral symmetry. Its first stellation is the compound of a cube and its dual octahedron, with the vertices of the cuboctahedron located at the midpoints of the edges of either.

The cuboctahedron is a rectified cube and also a rectified octahedron.

It is also a cantellated tetrahedron. With this construction it is given the Wythoff Symbol: **3 3 | 2**.

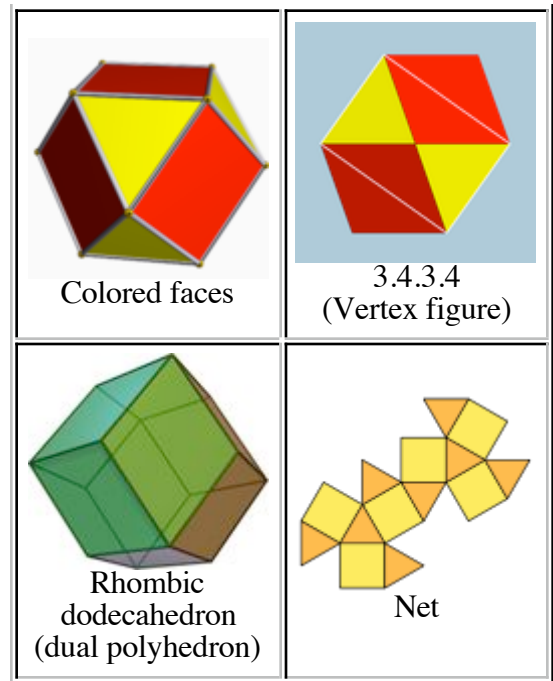
A skew cantellation of the tetrahedron produces a solid with faces parallel to those of the cuboctahedron, namely eight triangles of two sizes, and six rectangles. While its edges are unequal, this solid remains *vertex-uniform*: the solid has the full tetrahedral symmetry group and its vertices are equivalent under that group.

The edges of a cuboctahedron form four regular hexagons. If the cuboctahedron is cut in the plane of one of these hexagons, each half is a triangular cupola, one of the Johnson solids; the cuboctahedron itself thus can also be called a triangular gyrobicupola, the simplest of a series (other than the gyrobifastigium or "digonal gyrobicupola"). If the halves are put back together with a twist, so that triangles meet triangles and squares meet squares, the result is another Johnson solid, the triangular orthobicupola.

Both triangular bicupolae are important in sphere packing. The distance from the solid's centre to its vertices is equal to its edge length. Each central sphere can have up to twelve neighbors, and in a face-centered cubic lattice these take the positions of a cuboctahedron's vertices. In a hexagonal close-packed lattice they correspond to the corners of the triangular orthobicupola. In both cases the central sphere takes the position of the solid's centre.

Cuboctahedra appear as cells in three of the convex uniform honeycombs and in nine of the convex uniform polychora.

The volume of the cuboctahedron is $5/6$ of that of the enclosing cube and $5/8$ of that of the enclosing



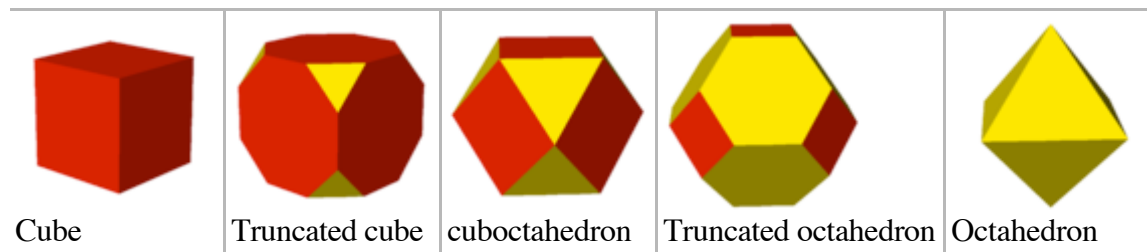
octahedron; it is $5/3 \sqrt{2}$ times the cube of the length of an edge.

Cartesian coordinates

The Cartesian coordinates for the vertices of a cuboctahedron (of edge length $\sqrt{2}$) centered at the origin are

$(\pm 1, \pm 1, 0)$
 $(\pm 1, 0, \pm 1)$
 $(0, \pm 1, \pm 1)$

Related polyhedra



See also

- Cube
- Icosidodecahedron
- Octahedron
- Rhombicuboctahedron
- Truncated cuboctahedron

References

- Williams, Robert (1979). *The Geometrical Foundation of Natural Structure: A Source Book of Design*. Dover Publications, Inc. ISBN 0-486-23729-X. (Section 3-9)

External links

- The Uniform Polyhedra (<http://www.mathconsult.ch/showroom/unipoly/>)
- Virtual Reality Polyhedra (<http://www.georgehart.com/virtual-polyhedra/vp.html>) The Encyclopedia of Polyhedra
- Eric W. Weisstein, *Cuboctahedron* (*Archimedean solid*) at MathWorld.

Retrieved from "<http://en.wikipedia.org/wiki/Cuboctahedron>"

Categories: Archimedean solids | Quasiregular polyhedra

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