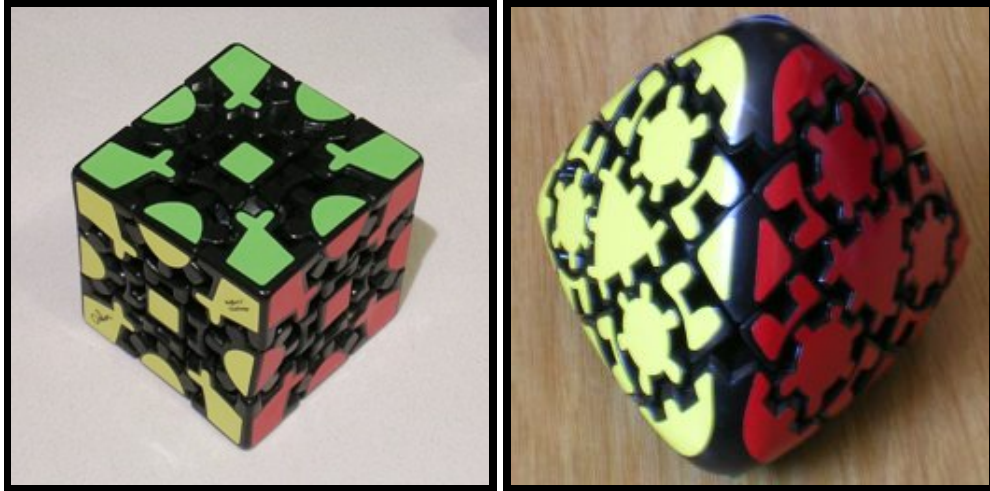# The Gear Cube / The Gear MasterMorphix

The Gear Cube was invented by Oskar van Deventer, based on an idea by Bram Cohen. It is like a 3×3×3 Rubik's Cube where all the edge pieces are cogs that turn when the outer layers are moved. If you give any face a half turn, the middle layer moves a quarter turn due to the edge cog wheels.

The puzzle is manufactured and sold by Uwe Meffert. It is quite an easy puzzle. On the standard version the edge pieces only have stickers on the rotating cog, but it is possible to put stickers on the non-rotating base part of the edge pieces to make it a little harder.

This puzzle should not be confused with the much harder Gear Cube Extreme, also known as the Anisotropic Gear Cube.

The Gear MasterMorphix is a tetrahedral version of the Gear Cube. It is slightly more difficult because the square pieces that are face centres on the cube are now edge centres, and so have two colours and visible orientation. In effect it is a 'supercube' variant of the fully stickered Gear Cube.

The number of positions:

The pieces of the gear cube are very restricted in their movements, because you can only do anti-slice moves. Instead of the centres, lets use a corner piece as a fixed reference point. The corners split into two tetrads, which do not mix. The locations of the three moving corners in the tetrad with the fixed reference corner are fully determined by the locations of the four corners in the other orbit. Those four other corners have 4! = 24 possible permutations.
The four edges in any slice will always remain in the same slice. Given a corner permutation, it turns out that the edges in a slice can only be permuted in 4 ways. They also all have the same twist, for which there are three possibilities. The three slices therefore contribute a factor $(4 \cdot 3)^3$ to the number of positions.
The centres can permute but this is fully determined by the edge permutation.
This gives a total number of positions of $4!(4 \cdot 3)^3 = 41,472$.

If the edge bases also have stickers on them, then their orientation becomes visible. There are only 4 possible orientation arrangements these can have, making the total number of positions $4!(4 \cdot 3)^3 \cdot 4 = 165,888$. Note that this is 27 times the number of positions in the cube anti-slice group because of the 3^3 possible twists of the three edge slices.

The Gear MasterMorphix can have its edge-centres twisted. It turns out that without affecting any other pieces, opposing edge-centres must be twisted the same amount, and can only be twisted by half turns. As there are three opposing pairs, that means that the total number of possible positions is increased by a factor of 2^3, to make $4!(4 \cdot 3)^3 \cdot 4 \cdot 2^3 = 1,327,104$.

I used my computer to calculate God's Algorithm for all three variations of the Gear Cube. Here are the results for the Gear Cube without stickers on the edge bases:

	Multiple turns							
Single turns	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**Total**
0	1							**1**
1		6						**6**
2		6	24					**30**
3		6	48	84				**138**
4		6	60	276	264			**606**
5		6	72	540	1,218	264		**2,100**
6		3	96	1,118	3,048	1,680	96	**6,041**
7			120	1,992	5,796	4,842	702	**13,452**
8			108	1,610	5,721	4,650	1,189	**13,278**
9			48	252	1,716	2,220	756	**4,992**
10			3	36	306	285	144	**774**
11				12		36		**48**
12				1	3		2	**6**
Total	1	33	579	5,921	18,072	13,977	2,889	**41,472**

This shows that if any number of turns of one face is considered as one move, then at most 6 moves are necessary. If each half turn of a face counts as one move, then 12 moves are sometimes needed.

The antipodes are (see below for notation)
U6 R6 F6 (a 6X pattern)
R3 U6 F6 R3 (a 2X+4H pattern, occurs in 3 orientations)
U3 F3 R6 U6 F3 U9 (a 6H pattern, occurs in 2 orientations)

Here are the results for the Gear Cube with stickers on the edge bases:

	Multiple turns									
Single turns	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Total**
0	1									**1**
1		6								**6**
2		6	24							**30**
3		6	48	96						**150**
4		6	60	276	384					**726**
5		6	72	492	1,218	1,128				**2,916**
6		3	96	878	3,048	3,168	2,715			**9,908**
7			120	1,632	4,872	11,382	6,024	2,724		**26,754**
8			108	1,850	7,101	14,562	17,987	7,629		**49,237**
9			48	756	4,152	14,556	18,870	7,920	2,832	**49,134**
10			3	36	1,182	4,869	8,159	8,529		**22,778**
11				12		276	1,620	1,020	1,044	**3,972**
12				1	3		2	258		**264**
13									12	**12**
Total	1	33	579	6,029	21,960	49,941	55,377	28,080	3,888	**165,888**

This shows that if any number of turns of one face is considered as one move, then 8 moves are necessary to solve the gear cube with extra stickers in the worst case. If each half turn of a face counts as one move, then 13 moves are sometimes needed.

Finally, the results for the Gear MasterMorphix, or the Gear supercube:

	Multiple turns											
Single turns	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**
0	1											**1**
1		6										**6**
2		6	24									**30**
3		6	48	96								**150**

ingle turns												Total
4		6	60	276	384							726
5		6	72	396	1,200	1,440						3,114
6		3	96	512	2,346	4,176	4,554					11,687
7			96	660	3,108	11,016	11,088	10,686				36,654
8			72	852	4,599	15,630	29,409	25,444	16,677			92,683
9			72	828	4,716	21,630	47,706	51,462	38,616	18,840		183,870
10			39	774	5,262	20,751	55,209	78,889	86,076	28,962	2,709	278,671
11				672	3,840	20,100	51,480	80,208	84,180	66,696	1,440	308,616
12				494	3,090	12,624	34,392	57,806	87,603	38,061	5,242	239,312
13				288	1,464	6,510	13,200	34,176	35,964	32,370	768	124,740
14				133	681	522	6,846	5,498	16,572	9,369	554	40,175
15				48		144	240	1,992	648	2,946	384	6,402
16								60	108	72	27	267
Total	1	33	579	6,029	30,690	114,543	254,124	346,221	366,444	197,316	11,124	1,327,104

So this puzzle can be solved in 10 moves, or 16 if each half turn of a face counts as a single move.

Links to other useful pages:

Uwe Meffert's pages. He produces and sells the Gear Cube as well as many other puzzles.

Notation:

Let F denote a clockwise half turn of the front face, keeping the rear face stationary. Similarly, let R and U denote clockwise half turns of the Right and Upper faces respectively.

Solution:

Phase 1: Solve the corners.
Note that on the Gear MasterMorphix this phase will solve the 4 corners and the 4 triangular face centres.

   a. Find two of the cube's corner pieces that should be adjacent (they have two colours in common). Concentrating on only those two corners, do any moves needed to make them match. This is easy and takes at most two moves.
   On the Gear MasterMorphix, find one face centre and one corner that share a colour and make them adjacent.
   b. The eight corners (corners and face centres on the MasterMorphix) now form four matching pairs. Hold the puzzle so that the matching pairs form vertical columns. Doing R or U keeps those columns intact, and by alternating those moves the columns will eventually all match up so that all eight pieces are correct.

Phase 2: Position the geared pieces.
In the cube these are the 12 edges, and on the tetrahedron these are the gears of which each face has three.

   a. Look at the geared piece at the UF location, lying at the top front in the vertical slice between the right and left layers. By looking at its colour(s), find out where in this slice the piece belongs.
   b. Do one of the following, depending on where the gear piece belongs:
   UF: Do nothing.
   UB: Do F R R F
   DB: Do R R
   DF: Do U R R U
   All four gear pieces in the vertical slice should now be correctly positioned.
   Note that the orientation of the gears and of the surrounding parts is fixed in the next phase.
   c. If any further gear piece is incorrectly positioned, then hold the puzzle with that edge piece at the UF location, and do steps a-b to solve it. Repeat until all are correct

Phase 3: Orient the pieces.

   a. If any gear piece needs to be twisted, then hold the puzzle with that gear piece at the UF location, and do R R R R. Repeat if necessary until all gears are twisted correctly.
   b. If the base parts on either side of a gear piece have visible orientation and some need to be flipped, then hold the puzzle so that the unflipped base parts lie in the horizontal middle layer, and do F R F R F R. Repeat if necessary until all base parts are oriented.
   c. If the square pieces have visible orientation, such as the edge-centres on the Gear MasterMorphix, and some need to be

flipped, then hold the puzzle so that the U centre is one such piece, and do F'2 R F R' U3 R F' R' F2 U3. Repeat if necessary until all parts are oriented.

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