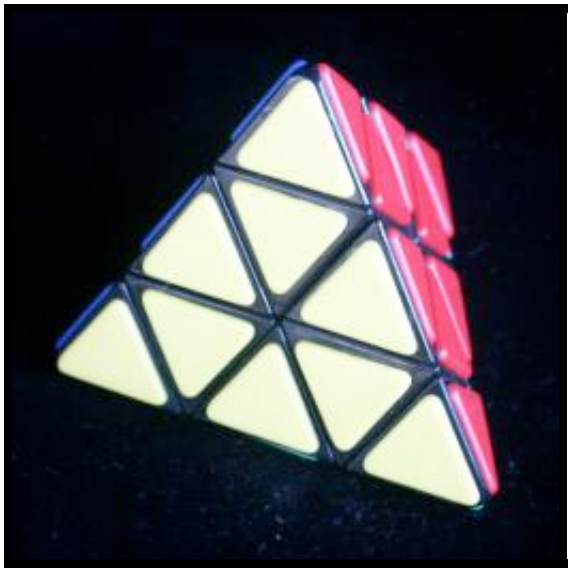
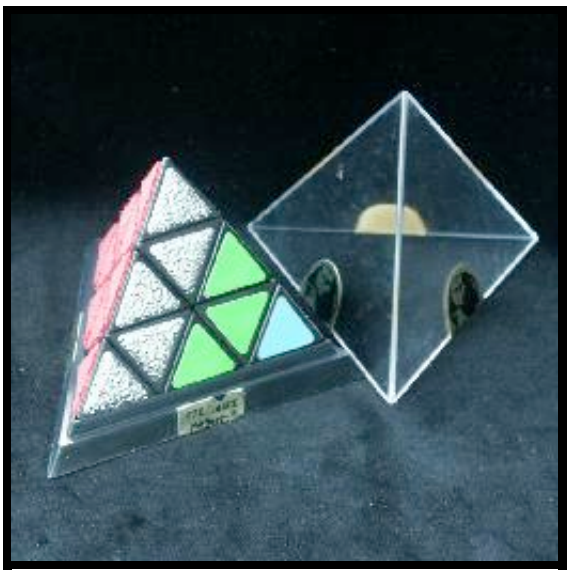
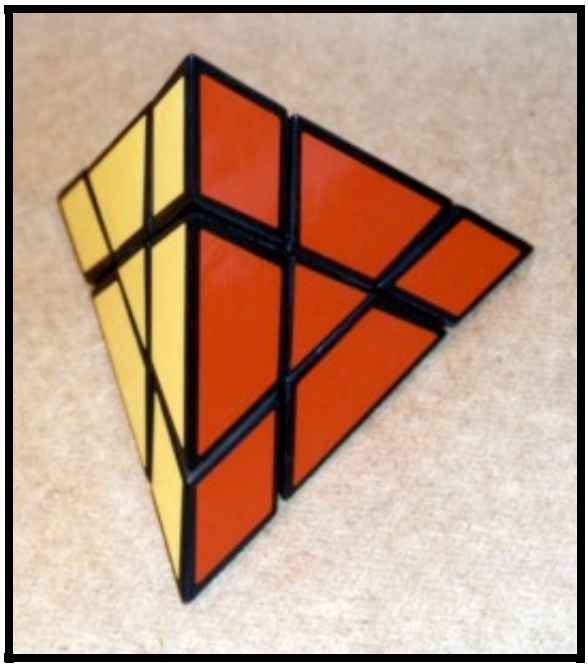
Jaap's Puzzle Page

Pyraminx / Tetraminx







Picture kindly supplied by Hendrik Haak

The standard Pyraminx is a regular tetrahedron, of which each triangular face is divided into 9 identical smaller triangles, three to a side. A move consist of rotating a sub-tetrahedron around a vertex; either just a tip or a larger tetrahedron (two triangles to a side). Each face has one colour, so the edge pieces have two colours, while the tips and the adjacent vertex pieces have three colours each.

The Pyraminx was invented by Uwe Mèffert in about 1971, but he only started producing it when the Cube Craze hit, and after he had carefully secured patents in most relevant countries, in particular Taiwan and Korea which haven't signed international patent treaties. A European Patent was granted on 30 December 1981, EP 42,695.

There several similar puzzles. The Tetraminx is a snub tetrahedron, which is the same as a Pyraminx but without the tips, also produced by Mèffert (though the first ones were crude versions made by Rubin's Toy Co. from Canada). A similar puzzle not pictured here was produced in Taiwan, and has a more rounded shape, with circular faces (like a sphere with 4 caps cut off). The most interesting variation was invented independently, and is nowadays called the Halpern-Meier Pyramid. It is cut differently than the pyraminx; the sub-pyramids are larger so that each face has a centre piece. Mechanically, it is those faces which are fixed to the centre, whereas in the pyraminx it is the corners which are on axes. Uwe Meffert bought the patent rights to it.

If your browser supports JavaScript, then you can play the Pyraminx by clicking the link below:

Javascript Pyraminx

The number of positions:

There are 4 vertex pieces with 3 orientations, 6 edge pieces with 2 orientations giving a maximum of $6!\cdot 2^6\cdot 3^4$ positions. This limit is not reached because:

- only an even number of flipped pieces are possible (2)
- only even permutations of edges are possible (2)

This leaves $6!\cdot 2^4\cdot 3^4$ = 933,120 positions.

If you include the trivial vertex tips then this has to be multiplied by a further 3^4=81. The Halpern-Meier pyramid instead has only 4 times as many positions.

This puzzle is closely related to the Skewb which also has four axes of rotation. The vertices and the face centres (if present) of the pyramid correspond to the corners of the Skewb, and the edges to the square faces of the Skewb.

A computer search gives the following result (ignoring the tips):

Depth	Pyraminx	H-M Pyramid
0	1	1
1	8	8
2	48	48
3	288	288
4	1,728	1,728
5	9,896	10,128
6	51,808	57,780
7	220,111	305,483
8	480,467	1,239,266
9	166,276	1,879,631
10	2,457	237,320
11	32	778
12		21
Total	933,120	3,732,480

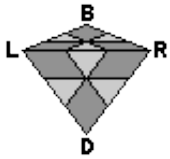
This shows that any pyraminx position can be solved in at most 11 moves, or 12 for the Halpern-Meier Pyramid. In Sloane's On-Line Encyclopedia of Integer Sequences these are included as sequences A079744 and A079746. See the Ultimate Skewb page for tables of all the Skewb-related puzzles.

The number of positions is relatively small, so a computer can very quickly search through all positions to find the shortest possible solution for any given position. If your browser supports layers/styles, (i.e. Netscape 4+, Explorer 5+) then you can play a Javascript version which includes such a solver. It takes a moment to load because it has to prepare some pruning tables for the search algorithm of the solver.

Links to other useful pages:

Uwe Mèffert's pages. He invented and now produces and sells the Pyraminx amongst others.
Meffert's page. Contains the solution provided in the Pyraminx booklet.
Meffert's page. Contains the solution provided in the Tetraminx booklet.
'Simplest Solutions' page. A text based solution.
Cyril Castella's page. A French solution.
Matthew Monroe's page. A nice illustrated solution.
Oxford Computing lab has a short text based solution here.
Chris Eggermont's page. A few useful move sequences which are enough to solve it.
A Nerd Paradise has solutions for the various cubes, Pyraminx, Skewb and Square-1.

Notation:



Hold the tetrahedron so that the vertices point left, right, down and back. Denote these by L, R, D and B. Anti-clockwise turns are denoted L', R', D' and B'. The faces are denoted F, U, L and R (front, up, left, right) so that edge pieces can be easily described by a combination of two of those letters.

Solution:

Phase 1: Solve the tips

a. Simply rotate each tip to match its adjacent vertex piece.

Phase 2: Solve the U vertices.

a. Look at vertex D and find out which colour is not present.
b. Rotate the three other vertices so that the colour is on the top face. Now the whole top layer is correct apart from the edge pieces.

Phase 3: Solve the U edges.

a. Find an edge at the D vertex which belongs in the U face.
b. Hold the puzzle so that the edge belongs at the FU position.
c. Rotate D to bring the piece to the front so that the side which has the same colour as the U face is not in the front face.
d. Use one of the following sequences to place the piece correctly:
   1. To move FR->FU, do LD'L'.
   2. To move FL->FU, do R'DR.
e. Repeat the above until the top face is correct. If necessary, you can use one of the above sequences to displace any incorrectly placed pieces.

Phase 4: Solve the D vertex piece.

a. Simply rotate D to make its corner match the others.

Phase 5: Solve the D edges.

a. There are now only a few possibilities left. Hold the puzzle so that one of the following sequences will solve the puzzle completely:
   1. To move FR->LF->RL->FR, do R'DRDR'DR.
   2. To move FR->RL->LF->FR, do R'D'RD'R'D'R.
   3. To move FR->RF, FL->LF, do DRD'LD'L'DR'.
   4. To move FR->FL->LR->FR, do RLDL'D'R'.
   5. To move FR->LR->FL->FR, do RDLD'L'R'.

Phase 6: Solve face centres, for the Halpern-Meier pyramid only.

a. To swap the F-U and L-R centres do R'D'RD R'D'RD R'D'RD.

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