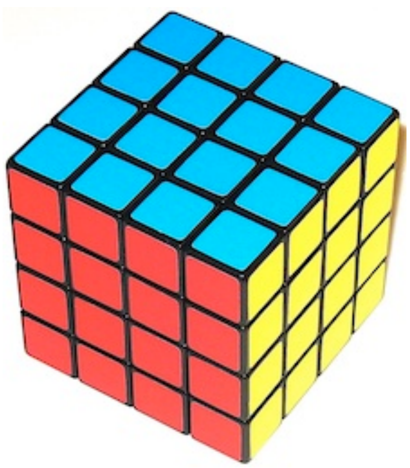
Rubik 4x4x4 "Revenge"

a.k.a. *Rubik's Master Cube*

"Rubik's Revenge";
Patented by P. Sebesteny 1983.
(plastic, 2.5 inches)

D-FantiX 4x4x4 Stickerless;
purchased from Amazon.com, 2017.
(plastic, 2.3 inches)

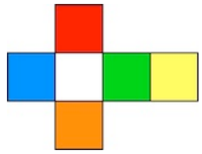


Mefferts "Rubik's Master" 2007;
uses the Eastsheen mechanism,
patented by C. Li 1999.
(plastic, 2.3 inches)

Newisland 4x4x4 Stickerless;
purchased from Amazon.com, 2017.
(plastic, 2.5 inches)

Rubik 4x4x4 Solution By Reduction to 3x3x3 Solution

Determine your cube color scheme: Since a Rubik 4x4x4 cube has no center squares, the first thing is to determine the color scheme of your cube (by looking at the corners and reasoning it out). Here is a diagram of the standard color scheme, which we use here:



Notation: We use the same notation as for Rubik's 3x3x3 with the additional convention that a lower case letter means move the corresponding row that is one in from that edge. For example, **U**, as with Rubik 3x3x3, means rotate the top clockwise 90 degrees, and **u** means rotate the layer below the top (the third layer up from the bottom) by 90 degrees clockwise. We use brackets to move two rows together; for example, **[Uu]** denotes rotating the top half of the cube 90 degrees clockwise. In fact, if it is a dexterity challenge to move a single middle layer, an easy way to do a **u** move, for example, might be **[Uu]** followed by **U-**.

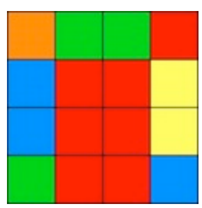
Basic Approach:

The following three pages described the three phases of solving:

Phase 1: Match up the centers.

Phase 2: Match up the edges.

After Phases 1 and 2 are completed, each face will have the correct color in the center four squares and, although the edges are in general completely mixed up, they are all in matching pairs; that is a typical face, such as the red face, might look like this:



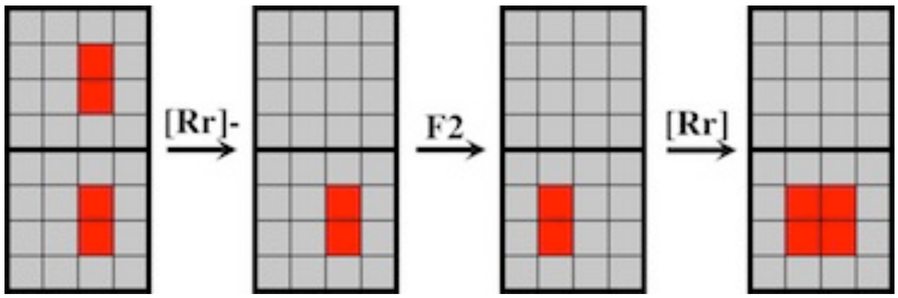
Phase 3: Solve like a Rubik's 3x3 cube.

Normal Rubik 3x3x3 moves never mess up the center groups or edge pairs (the pairs move around just like single edges do on a Rubik 3x3x3 cube). Solve using any layer at a time Rubik 3x3x3 solution.

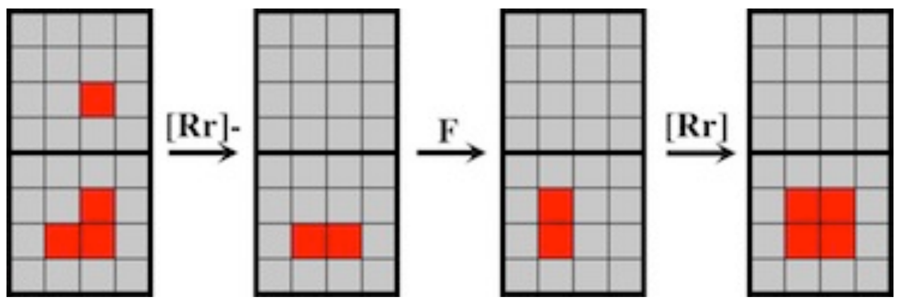
It may be that after solving, there remains a single edge pair flipped and / or exactly two corners or edge pairs exchanged (which cannot happen with a standard Rubik 3x3x3 cube), and a little additional work is done to finish up.

Phase 1 - Solve the Centers:

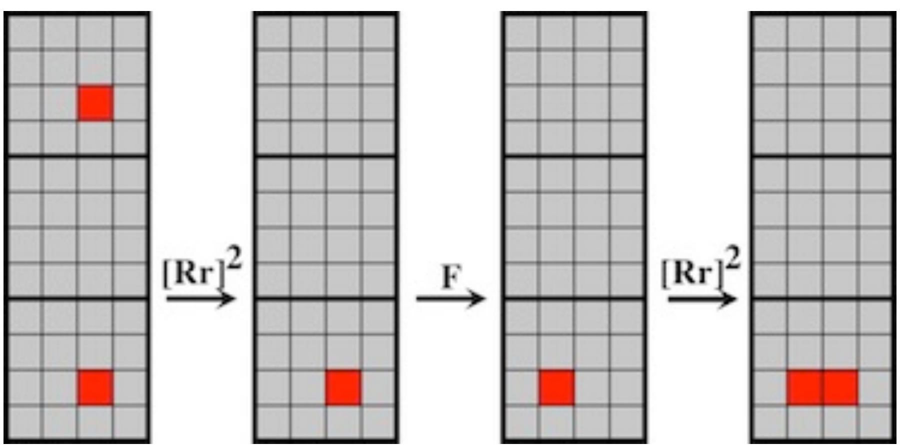
This can be done one face at a time (in any order) with a variation on a simple 3 move sequence. For example, if there are a pair of red squares on the front face and also a pair on the up face, then the faces can be oriented so that the pairs line up, and then do:



Or suppose that the front face already has three red squares and the fourth one is on the top face:



Or suppose there is a red square on the back that needs to come to the front.

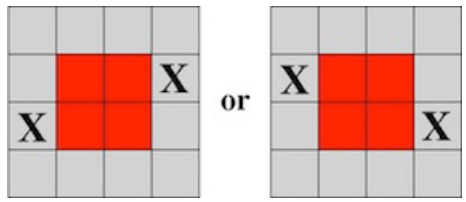


All three of these examples are essentially the same simple idea:

> Orient the cube so that the face is on front and what you want to move to the front is on the up face or the back face, then rotate the two faces so that that after the first step you will not have two squares diagonally opposite, and do:
>
> 1. Rotate the right half of the cube as appropriate (or left half also works).
> 2. Rotate the front as appropriate.
> 3. Rotate the left or right half back.

Phase 2 - Pair up the edges:

Edges can be paired one at a time (in any order) by first placing two to be paired in either of the two positions shown here

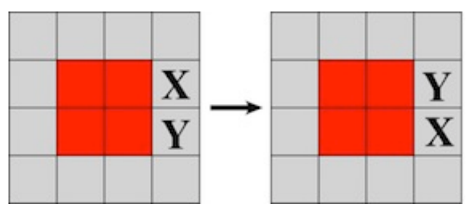


then doing **[Dd]** if they are positioned as on the left or **[Uu]** if they are positioned as on the right, then doing the simple three move sequence

R U R-

and finally doing a **[Dd]-** or **[Uu]-** to restore things. The net effect is to move XX to the up-front position, and to split the pair in either the up-back position or the up-right position. So before doing this put an unmatched pair in the up-back position if it is the case on the left above or the up-right position if it is the case on the right.

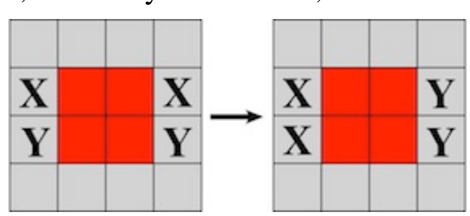
At first things are slow when one repeatedly hunts for a pair of matching squares to move them into position. If when you get them in position they are not offset, it is easy to fix that; e.g.:



R U- B- R^2

It is not necessary to memorize this sequence; regular 3x3x3 moves never un-pair edges, and just playing around suffices to move squares around. However, it saves time to remember it.

Finally, all edges will be paired, or exactly two are not, which can be paired with this sequence:



[Dd] R F- U R- F [Dd]-

This sequence is pretty easy to remember because of its symmetry, the **[Dd]** and **[Dd]-** at each end with a **U** in the middle, and it is **RF-** to the left of the **U** with the same to the right of the **U** except signs flipped to be **R-F.**

Phase 3 - Solve for Rubik 3x3x3 (and fix parity as needed)

Step 3A: Solve using any layer at a time Rubik's 3x3x3 Solution.

*** At this point if you are *lucky*, the cube is completely solved!

However, there is a 50-50 chance of each of two problems:

PLL parity: Exactly two corners or two edges are exchanged.

OLL parity: A single edge is flipped.

Step 3B: Fix OLL parity if present.

A complex sequence shown on the following page can be employed. However, OLL parity can be fixed as follows with having to memorize anything new:

A. Do a single *r* move.

B. Use Phases 1 and 2 to fix the affected centers and edge pairs.

C. Solve as for Rubik 3x3

Note: It doesn't matter how you hold the cube when you do Step A; the flipped edge can be in the front up position or any other position. Re-solving is much less work; the left and right faces are not affected, and the front top, back, and bottom faces all have the simple pattern of 2 of the correct color and 2 of the color from the adjacent face. For example, fix the front face by doing an **F2** followed by the standard **[Rr]**$^{-}$ **F2** **[Rr]** sequence of Phase 1, and repeat for the top and back face (which also fixes the bottom face). Now just a few edges need to be paired up again using Phase 2.

Step 3C: Fix PLL parity if present.

Starting with the cube fully solved except for two exchanged edges or two exchanged corners on the up layer, this sequence will fix PLL parity leaving the bottom three layers still solved, and so all that is left is to re-solve the up layer (in fact, if it is a front back edges exchange, the up layer will also end up fully solved):

$$\mathbf{r}^2 \quad \mathbf{U}^2 \quad \mathbf{r}^2 \quad [\mathbf{Uu}]^2 \quad \mathbf{r}^2 \quad \mathbf{u}^2$$

This sequence is relatively easy to remember as three **r2** moves, each followed by a variation of an up two move; think of the up moves as going inward, first **U2** (the outer face), then **[Uu]2** (the outer face and inner slice), and lastly **u2** (just the inner slice).

OLL Parity - Why Step 3B Works

OLL Parity is the number of *slice moves*, a 90 degree turn of a single inner layer. For example both **r** and **[Rr]** perform a slice move, but **[lr]** does not be because it comprises two slice moves, **l** and **r**. It is when OLL parity is odd that we end up with a single edge flipped, something that would be impossible for a 3x3x3 cube. Here "fixing" OLL parity refers to reversing OLL parity from odd to even in order for the cube to be fully solvable.

The basic [**Rr**]**-** **F2** [**Rr**] sequence of Phase 1 and the basic **[Dd] R U R- [Dd]-** or **[Uu] R U R- [Uu]-** sequences of Phase 2 do slice moves in pairs (**[Dd]**, **[Dd]-** or **[Uu]**, **[Uu]-**), and so Phase 1 and Phase 2 (done in the normal way) do not change OLL parity. So after Step 3B begins with a single slice move to change OLL parity, OLL parity is not further changed. And Step 3C does not change it either since it performs an even number of slice moves.

So in Step 3B, after the single **r** move is done, it doesn't matter exactly how the cube is repaired so long as all slice mores are done in pairs (which is true of the straightforward implementations of Phases 1 and 2). Just don't try to get fancy and slip in an extra slice move.

It could be that Step 3B changes PLL parity. That's ok; Step 3C will fix it if needed.

Directly Solving OLL Parity

Step 3B avoided having to memorize anything new to fix OLL parity. But if you like memorizing long sequences, this is a faster way to do it. Start with the cube solved as much as possible with the single flipped edge in the *front up position*, and use this sequence to correct that flipped edge and leave the rest of the cube unchanged (i.e., fully solved unless there is still an OLL parity problem to be fixed). Commas and the extra spacing have no meaning other than to help reading.

$$\mathbf{r^2\ B^2,\quad U^2,\quad l,\quad U^2\ r\text{-}\ U^2\ r\ U^2,\quad F^2\ r\ F^2,\quad l\text{-},\quad B^2\ r^2}$$

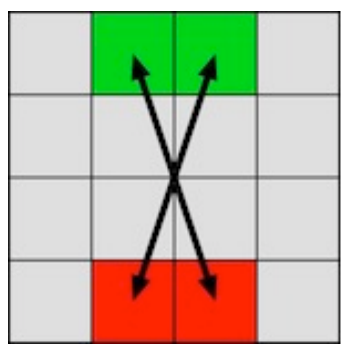
To make it easier to manipulate the cube, **r** and **l** can be replaced by **[Rr]** and **[Ll]**. The resulting sequence shown below also fixes OLL parity, but requires re-solving the up layer. This resolving does not amount to additional work if you identify the OLL parity problem before solving the up layer (it is present if the number of the up edge pairs correctly flipped is 1 or 3).

$$\mathbf{[Rr]^2\ B^2,\quad U^2,\quad [Ll],\quad U^2\ [Rr]\text{-}\ U^2\ [Rr]\ U^2,\quad F^2\ [Rr]\ F^2,\quad [Ll]\text{-},\quad B^2\ [Rr]^2}$$

Further Reading

The terms PLL and OLL parity are standard in the Rubik's cubing literature. Parity is just a property of a current cube state that is even or odd. OLL parity is the number of 90 turns of an inner layer (a slice move **r**, **l**, **f**, **b**, **u**, or **b**), where at the end an even number is good and an odd number results in a flipped edge. PLL parity refers to a relationship between edge and corner permutations; problems can arise with even dimenson cubes where there is no center square to guide you (see also, for example, the *Rubik 3x3x3 Void Cube* or using a Rubik 3x3x3 solution to solve a Rubik 2x2x2 cube). Here the references to "fixing" OLL and PLL parity are just informal ways of saying that the corresponding parity needs to be reversed from odd to even in order for the cube to be solvable.

Although these parities always remain even when solving a standard Rubik 3x3x3 cube, they can become odd during the solving of a 4x4x4 cube because there is more than one visually equivalent way to configure identical pieces when forming center groups or edge pairs. For example, the figure below shows how the cube could be fully solved except for exactly two edge pairs that are exchanged (an example of a PLL parity problem). This is impossible for a standard Rubik's 3x3x3 cube but in the 4x4x4 cube, each of the two halves of two edge pairs can be exchanged independently.



The complex sequences presented on the preceding page for directly solving OLL parity are from *Hardwick's Page*. There is much written in general about solving Rubik's 4x4x4 (and larger cubes) by reduction to 3x3x3 solving. There are also completely different approaches (e.g., layer by layer).

References

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