

Square 1



a.k.a. Super Cubix, Cube 21

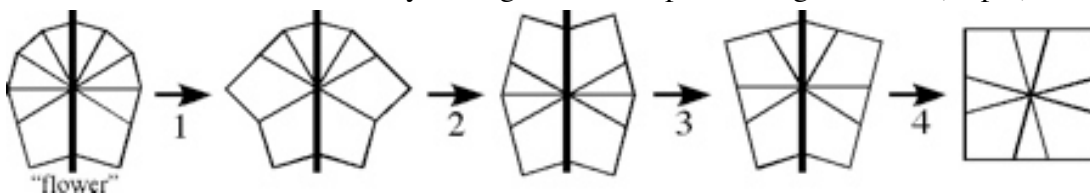
Patented by K. Hrsel and V. Kopsky 1993, copyright Irwin Toys 1990.
(plastic, 2.1 inches)

Three layers that form a cube when solved. The middle has two identical trapezoid pieces that can be in only one of two states, *square* or *nonsquare*. The top and bottom each have four 60 degree corner pieces and four 30 degree edge pieces.

The only moves are to *rotate* the top or bottom layers by a multiple of 30 degrees and to make a vertical 180 degree *twist* along the *reference plane* that passes through the two middle pieces.

The middle can be solved independent of top and bottom. Repeating three times a twist followed by a 180 degree rotation of the top changes the state of the middle (if you don't care about disturbing the top and bottom, a single 30 degree rotation followed by a twist suffices). A twist followed by 180 rotations of the top and bottom, followed by a twist flips the middle upside-down. And we can rotate the top and / or bottom after a sequence of moves to align them with the middle. Hence, we address solving the top and bottom, without worrying about the middle.

The easier problem of getting the puzzle back to a cube shape, but with colors in any arrangement (as in the second figure above) requires no explicit memorization. Gather the 8 edges in the top to form the "flower" pattern; the figure below shows the flower on the left and patterns resulting from 4 twists. The bottom layer below the flower has all 8 corners arranged in a star, however, the four positions resulting from each of the twists all have identical patterns on the top and bottom. So the rule for this sequence is "rotate along the major axis of symmetry 4 times" (shown by the dark lines). The only exception is when a twist gives you the same pattern back; in this case, rotate the bottom by 90 degrees before performing the twist (step 3).



Making the flower is not hard. Pairing edges and then combining pairs is the general approach, but you typically cannot avoid having two "stragglers" that are not paired. The key is to get a pair of stragglers separated by exactly two corner pieces; then you can park them in the bottom left, use rotations of the top and twists to get the other 6 edge pieces together, and then twist up the stragglers to become the first and last of the 8. If you get stuck with two stragglers separated by only one corner piece, split a group and rearrange, sooner or later things will work.

Solving Square 1

We combine approaches of *Jaap*, *Vandenbergh*, and *Nerd Paradise* for a solution of more moves, but requires only 3 non-trivial sequences to be memorized.

Holding the puzzle: Hold in your left hand with *Square 1* on the left, your thumb on the short orange section of the front of the middle layer and your finger on the long red section of the back of the middle layer. All twists are relative to the *reference plane* that goes from the front left to the back right.

Notation: Pieces are denoted by direction U (up), D (down), L (left), R (right), F (front), B (back); UFL corner, BR edge, etc. For moves, / denotes a twist and (x,y) means rotate the top x units and the bottom y units; a positive number is clockwise and a negative number is counterclockwise. For example,

change state of middle layer: / (6,0) / (6,0) /
flip middle layer: / (6,6) /

Step 1: Make the puzzle into a cube, as described on the previous page.

Step 2: Get all the white corners to the top and all the green corners to the bottom (but not necessarily in their correct locations).

If you are at the level that you can make it into a cube, you can easily do this. Get two green together and park them in the lower left. The using rotations of the top and twists get the other two greens together and twist them down.

Step 3: Get all the white edges to the top and all the green edges to the bottom (but not necessarily in their correct locations), using the transformation UB \leftrightarrow DF:

$(0,-1) / (-3,0) / (4,1) / (-4,-1) / (3,0) /$

A note on using the puzzle upsidedown: The moves in Steps 4 and 5 are defined for the top layer. To apply them to the bottom layer, turn the puzzle upsidedown, hold the puzzle with *Square 1* on the left (your thumb on the long red section on the front and your finger on the short orange section on the back) and precede a move sequence with $(-1,1)$, which restores the reference plane to effectively go from front left to back right.

Step 4: Position corners using the transformation UFL \leftrightarrow UFR:

$/(3,-3) / (3,0) / (-3,0) / (0,3) / (-3,0) /$

* Use three times to swap diagonally, or do that directly by using:

UBL \leftrightarrow UFR: $/(3,3) / (3,0) / (3,3) / (3,0) / (3,3) /$

Step 5: Position edges using the transformation UF \leftrightarrow UR:

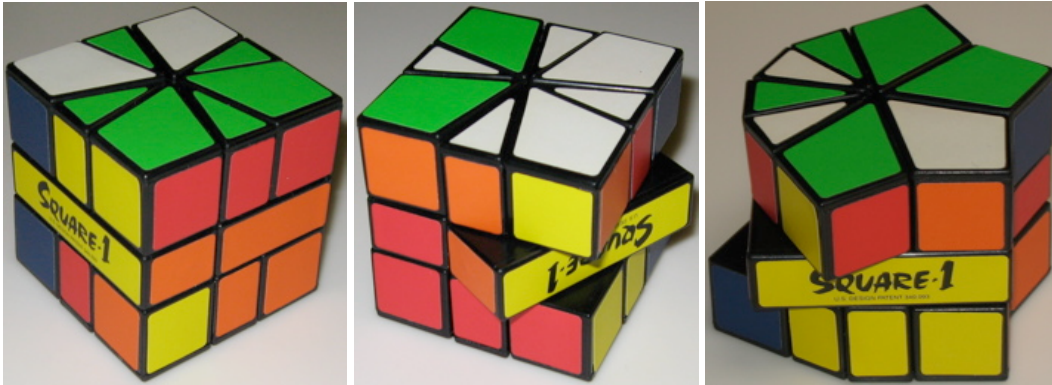
$/(-3,0) / (0,3) / (0,-3) / (0,3) / (2,0) / (0,2) / (-2,0) / (4,0) / (0,-2) / (0,2) / (-1,4) / (0,-3) /$

* Use three times to swap opposites, or do that directly by using:

UL \leftrightarrow UR: $/(3,3) / (-1,0) / (2,-4) / (4,-2) / (0,-2) / (-4,2) / (1,-5) / (3,0) / (3,3) /$

Step 6: Fix the middle layer if necessary.

Some Photos of Square 1 Mixed Up



Other Useful Sequences (that can be used for shortcuts)

Exchange the UF and UB edges with DF and DB (Nerd Paradise "Sequence A"):

$$(1,0) / (-1,-1) / (0,1)$$

Exchange two edge corner pairs (Nerd Paradise "Sequence B"):

$$DL \leftrightarrow DR, DLB \leftrightarrow DRB: / (3,0) / (-3,-3) / (0,3) /$$

Exchange the 4 top edges with the 4 bottom edges:

$$(1,0) / (-1,-1) / (-2,-2) / (-1,-1) / (0,1)$$

Exchange two pairs of corners (from Jaap's page):

$$UFR \leftrightarrow UBL, DFR \leftrightarrow DBR: / (3,0) / (-3,0) / (3,0) / (3,0) / (6,0) /$$

$$UFR \leftrightarrow UBR, DFL \leftrightarrow DBR: / (0,-3) / (0,3) / (0,-3) / (0,-3) / (0,6) /$$

Exchange two pairs of edges (from Jaap's page):

$$UF \leftrightarrow UB, DF \leftrightarrow DB: (1,0) / (-1,-1) / (6,0) / (1,1) / (-1,0)$$


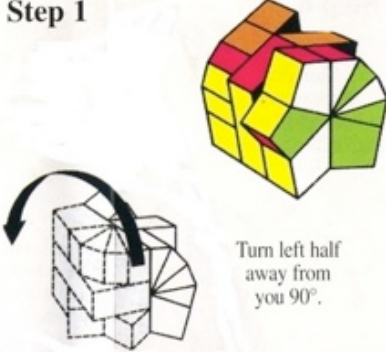
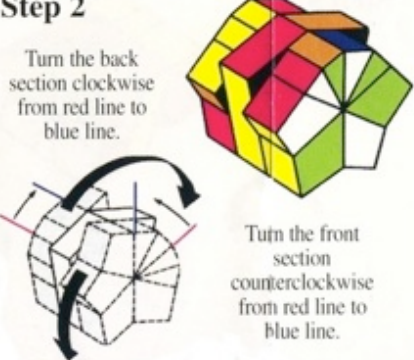
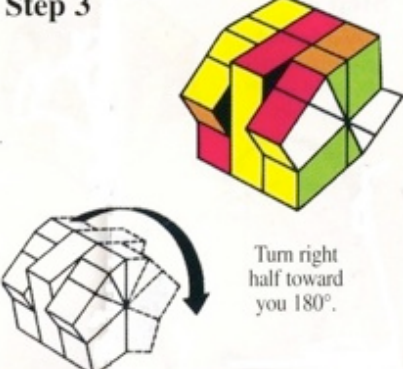
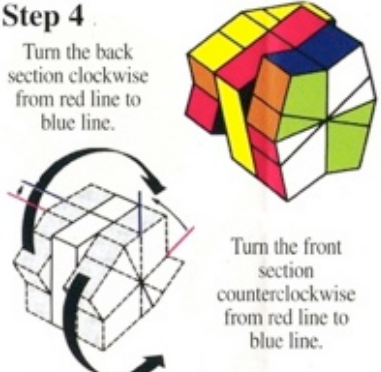
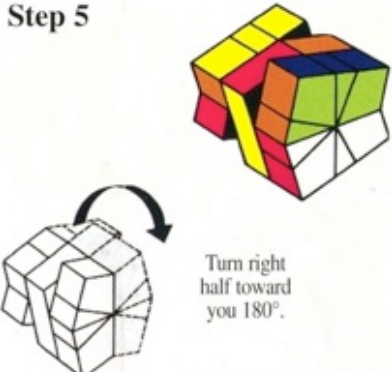
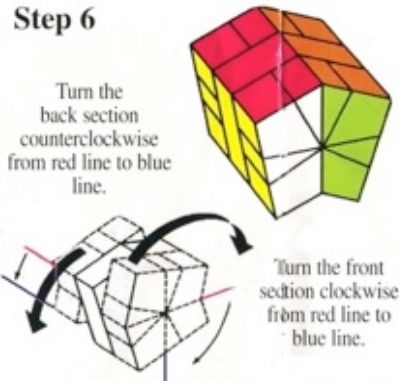
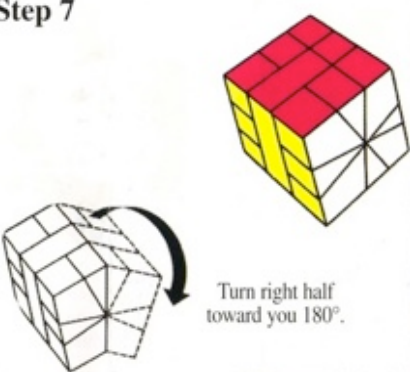
$$UL \leftrightarrow UF, DR \leftrightarrow DF: (1,0) / (-3,0) / (-1,-1) / (3,0) / (1,1) / (-3,0) / (-1,-1) / (4,1) / (-1,0)$$

Square 1 Package



Unpacking Square 1:

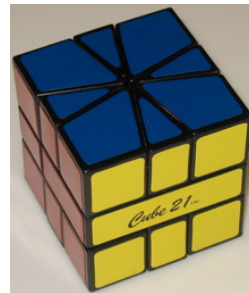
Square 1 is shipped mixed up in a way that looks intimidating but can be solved with 7 twists that are provided in the directions:

 <p>To begin- Hold the puzzle like this.</p>	<p>Step 1</p>  <p>Turn left half away from you 90°.</p>
<p>Step 2</p> <p>Turn the back section clockwise from red line to blue line.</p>  <p>Turn the front section counterclockwise from red line to blue line.</p>	<p>Step 3</p>  <p>Turn right half toward you 180°.</p>
<p>Step 4</p> <p>Turn the back section clockwise from red line to blue line.</p>  <p>Turn the front section counterclockwise from red line to blue line.</p>	<p>Step 5</p>  <p>Turn right half toward you 180°.</p>
<p>Step 6</p> <p>Turn the back section counterclockwise from red line to blue line.</p>  <p>Turn the front section clockwise from red line to blue line.</p>	<p>Step 7</p>  <p>Turn right half toward you 180°.</p>

Other Versions of Square 1



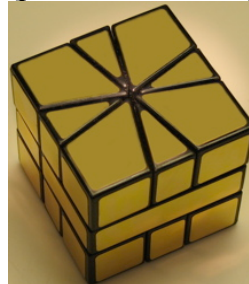
Super Cubix
Sold in U.K, circa 1990.
(plastic, 2.1 inches)



Cube 21
Sold in Hungary, circa 1990.
(plastic, 2.1 inches)



Cube 21 Silver
3 yellow sides with silver on opposite sides.
(plastic, 2.1 inches)



Cube 21 Gold
Custom made - all gold.
(plastic, 2.1 inches)



Square 1 Black
All black, purchased from Cubfans 2007.
(plastic, 2.1 inches)



Square 1 Clear
All clear, purchased from Cubfans 2007.
(plastic, 2.1 inches)

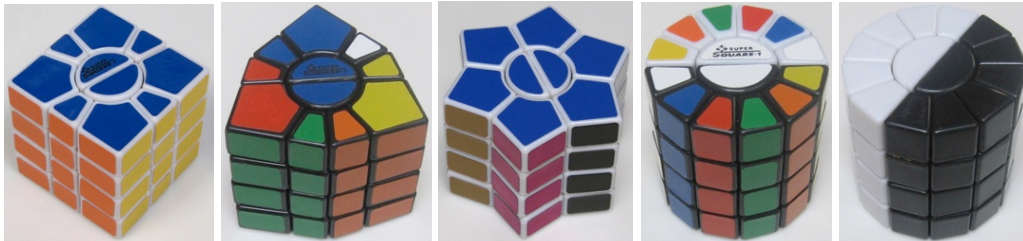


Zhaofeng Square 1
Made in China, purchased 2005.
(plastic, 2.1 inches)

Square 1 Variants



Left: "Square 2", purchased from Mefferts 2012.
Right: "Dodecagonal Barrel", custom made by Laurent Langlade 2007.
(both plastic, made with Square 1 parts; 2.2 inches;
top and bottom layer can each rotate by multiples of 30 degrees
and a vertical 180 twist can be made along a single reference plane)



"Super Square 1", made by Smaz Smart Toy Shop IQ Toys, Hong Kong, 2008-2009.
(plastic, 2.2 inches high;
extra center layer where everything rotates around a center post that splits;
switching the top and bottom layers will not change the color of the top of the posts.)



"Two-Layer Square 1", made in China 2011.
(plastic, 2.2" x 2.2" x 1.1", a Super Square 1 without the middle layers)

Further reading:

Jaap's Page - Square 1, from: <http://www.jaapsch.net/puzzles/square1.htm>

Jaap's Page - Square 2, from: <http://www.jaapsch.net/puzzles/square2.htm>

McFarren's Page, from: <http://www.geocities.com/abcmcfarren/math/sq1/sq1xf.htm>

McFarren's Page2, from: <http://www.geocities.com/abcmcfarren/math/sq1/sq1xf.htm>

Alchemistmatt Page, from: <http://www.alchemistmatt.com/cube/square1.html>

Dry Erase Board Page, from: <http://www.thedryeraseboard.com/mechpuz/square1/solution>

Nerd Paradise Page, from: <http://nerdparadise.com/puzzles/square1>

Vanderber's Page, from: <http://www.cubezone.be/square1.html>

Monroe's Page, from: <http://www.alchemistmatt.com/cube/square1.html>

Eggermont's Page, from: <http://web.inter.nl.net/users/C.Eggermont/Puzzels/SquareOne>

Eggermont's List of all Square 1 configurations, from:

<http://web.inter.nl.net/users/C.Eggermont/Puzzels/SquareOne/SquareList.html>

F2 Page, from: <http://f2.org/math/square1>

Hrsel and Kopsky Patent, from: www.uspto.gov, patent no. 5,193,809