Rhombic Polyhedra Puzzles Part 2

by George Bell and Stephen Chin

Introduction

In Part 1 of this article series [1], we introduced the basic geometry of Golden Rhombohedra, which we nicknamed **rhombs**. Rhombs are 6-sided polyhedra where the ratio of the diagonals (on every face) is equal to the golden ratio. Rhombs come in two varieties, **fat** (**F**) and **thin** (**T**), and they fill space.

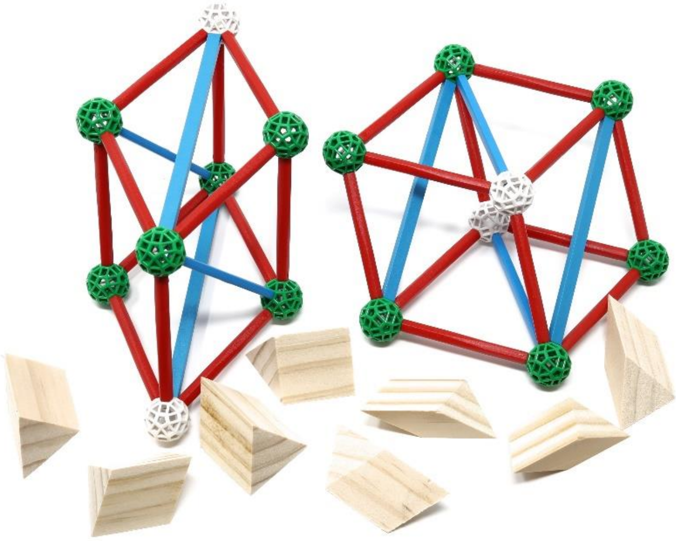

Figure 1. Zometool [2] Rhombs split into identical or mirror image halves. Foreground: wood building blocks for split-rhomb puzzles.

There are many ways to cut rhombs in half, but Wayne Daniel chose a particular dissection of each rhomb which we now focus on. The parallel blue struts in Figure 1 show how each rhomb can be cut into identical or mirror image halves. Interestingly, Wayne Daniel chose the symmetrical decomposition for the fat rhomb and the mirror image decomposition for the thin rhomb (using the shorter blue struts in both cases). For the fat rhomb, the newly created face appears square. But by careful calculation of the vertex coordinates one can show that this new face is almost, but not quite square.

In Part 1 we discussed puzzles made from complete rhombs. Part 2 was reserved for split-rhomb puzzles. However, after Part 1 was published we discovered some new designs using full rhombs which we will first summarize.

Puzzle Geometry

Part 1 generated much interest in triacontahedron puzzles, especially among the authors. One thing that interested us was the “spoon” piece, made from 3 fat and 3 thin rhombs. Three spoons go together with coordinate motion to form a triacontahedron minus the N Pole and core. Was this spoon piece the only piece with this property?

Quickly a small program was created to search for all such pieces. The details of the triacontahedron geometry (i.e. vertex coordinates) were not needed by this program. Removing the N pole and core, we are left with 18 rhombs arranged with 3-fold symmetry about a central axis. The nine fat rhombs can be separated into three sets of triples, where the sets define fat rhombs equivalent by symmetry (120° rotation). Likewise, the nine thin rhombs can be separated into three more sets of triples.

To create a piece, we choose one rhomb from each of the six sets, resulting in $3^6 = 729$ pieces. Most of these pieces are not connected. In order to determine if a piece is connected, the program needs the adjacency matrix of rhomb interconnections. A quick run produced the answer: there were 22 unique connected pieces with the property that three of them form the triacontahedron (minus the core and N pole).

So far, two new coordinate motion puzzles have been found (Figure 2). These we have designated GBRT04 and GBRT05. The yellow piece can be identified as a "Batman" piece with two rhombs added, the blue piece is interesting because it contains a hole. For a complete description of these puzzles, see the supplementary material for this CFF article.

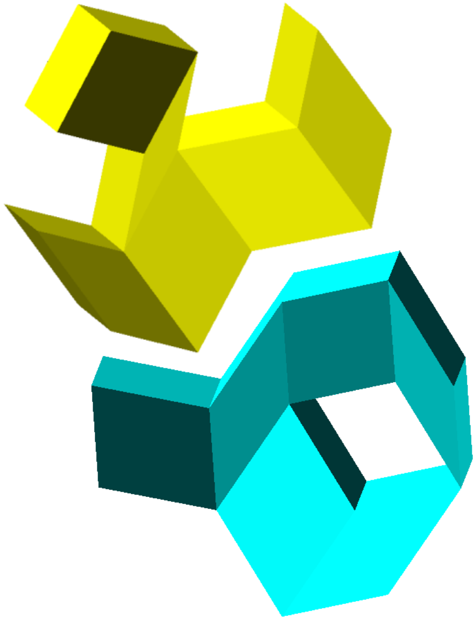

Figure 2. The basic piece for new puzzles GBRT04 and GBRT05.

Dodecahedron Puzzles from Split-Rhombs

The Bilinski Dodecahedron is composed of two fat and two thin rhombs, and would seem a good candidate for a split-rhomb puzzle. Stephen Chin created a two piece dissection using six hybrid sub pieces (two full rhombs plus four split-rhombs). The *Bilinski Dodecahedron* was his IPP 39 Exchange puzzle in Kanazawa Japan 2019, see Figure 3.

Figure 3. *Bilinski Dodecahedron* by Stephen Chin (2019).

Icosahedron Puzzles from Split-Rhombs

Wayne Daniel created these puzzles in the 1980's and sold them through his company **Interlocking Puzzles**. When assembled these puzzles appear identical (Figure 4), and Wayne Daniel was careful to make each design out of a specific wood. In hindsight, this was a great help to collectors, because one can identify the puzzle by the wood type, and it makes it easy to tell when you have a complete set. We continued this tradition in our 3D printed copies by always dyeing each puzzle a specific colour.

Figure 4. James Dalgety's complete collection of Wayne Daniel Icosahedra. Left to right: WDRI51, 55, 53, 50, 52, 54, 56. Photo courtesy puzzlemuseum.org [4].

The seven designs are listed in Table 1, all these puzzles have 4 pieces and are serially interlocking. We've included two identifiers for each puzzle. Our ID was explained in Part 1—we number starting at 50 for puzzles using split-rhombs. The Original ID is printed (or written) on the original box. We also started naming the puzzles based on the shape of the first piece to be removed. For all Wayne Daniel wood puzzles (Figures 4 and 6) the scale factor *s* (defined in Part 1) is 9.7 mm. I believe the edge length was designed to be exactly 30 mm.

Each piece is made from 5 half-rhombs (never a full-rhomb), and when we say a piece is split "3/2" it means the piece is made from 3 fat half-rhombs and 2 thin half-rhombs. In the seven designs there are three types of puzzles:

x) Two pieces are split 3/2, and the other two pieces are split 2/3.
y) Three pieces are split 3/2 and the remaining piece is split 1/4.
z) Three pieces are split 2/3 and the remaining piece is split 4/1.

Our ID	First piece	Type/ Set	Orig. ID	Wood used	3D print colour; comment
WDRI50	Chinese A	y / B	#151	Gonçalo Alves	red
WDRI51	Chinese B	y / A	#124M	Bloodwood	blue
WDRI52	Kermit	x / B	#523	Cocobolo	yellow
WDRI53	Penguin	x / A	#8M	Red Heart	green
WDRI54	Slit-Slot	y / A	#1025M	Peroba Rosa	black; has 2 identical pieces
WDRI55	Swan B	z / B	#739S	Zebrawood	purple; rounding required to separate the final 2 pieces.
WDRI56	Swan A	z	#R800	Chechen	brown; IPP17 exch. puzzle

Table 1. The seven Wayne Daniel Icosahedron puzzles.

James Dalgety shared photos of his collection (Figure 4). He also saved original letters from Interlocking Puzzles. From these we discovered that these puzzles were sold in two sets of three (A & B), as indicated in Table 1. If you find three puzzles for sale they may be an original set. The seventh design (WDRI56) was Abel Garcia's exchange puzzle at IPP17 (1997) and produced last. A set of 3D printed puzzles is shown in Figure 5.

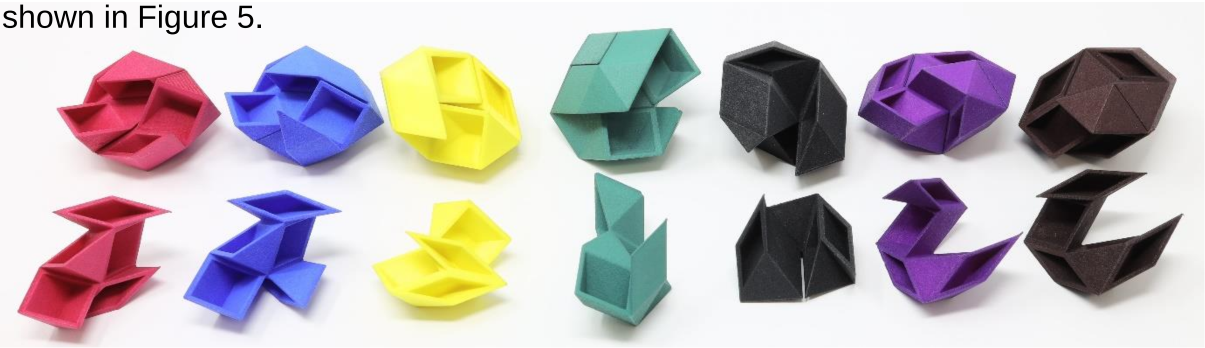

Figure 5. *WDRI50-56* 3D-printed by Shapeways (s=5.73 mm).
The first piece has been removed to show its shape.

One humorous story involves the labelling on the original boxes. On the box for WDRI50 (Figure 4), the label reads: "Design #151 of Goncalo Alves by Interlocking Puzzles." One of the authors (not so familiar with wood types) sent a "reply-all" asking "Whatever became of the famous puzzle designer Gonçalo Alves??"

Finally, Albert Gübeli has designed an icosahedron puzzle using fat rhombs split into four parts. *Sabine's Twist* [3] has 6 pieces, 5 of them form a rhombic icosahedron. This puzzle was entered in the IPP29 Puzzle Design Competition in 2009.

Triacontahedron Puzzles from Split-Rhombs

Wayne Daniel also created these puzzles in the 1980's. Several have been sold in recent auctions for prices as high as $750 (US dollars) each! We have found six Wayne Daniel puzzles, summarized in Table 2. We are uncertain at this time whether each puzzle was always made from a different wood type, like the icosahedra. In addition, there are no ID numbers on the original boxes, each design is identified only by the number of pieces.

Figure 6. James Dalgety's collection of Wayne Daniel Triacontahedra (s=9.7). Left to right: WDRT50, 53, 54, 55. Photo courtesy puzzlemuseum.org [4].

In order to make a 3D model of each puzzle, one must first figure out which of the four decompositions of the triacontahedron is used. We discovered that this can be accomplished by careful observation of the wood grain on the assembled puzzle.

First, look for faces where the grain doesn't match any of the four neighbouring faces (this includes faces which are cut in half). We put a blue sticker on each of these faces, there should be nine of them—one for each external fat rhomb.

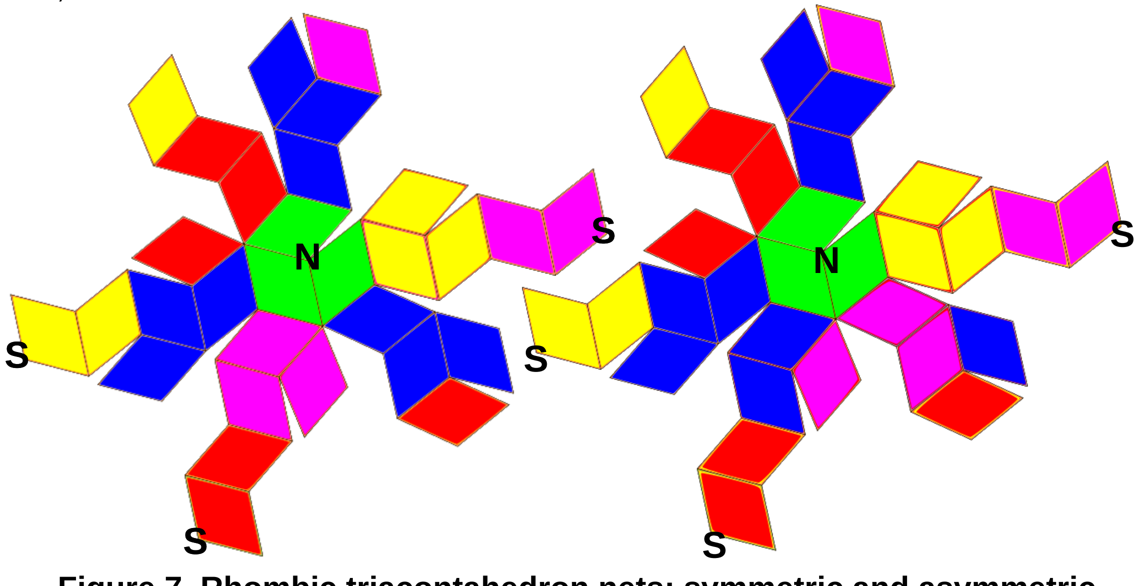

Figure 7. Rhombic triacontahedron nets: symmetric and asymmetric. N Pole rhomb (green), thin rhombs (red, yellow, pink), fat rhombs (blue).

Figure 7 shows two possible patterns on the faces of a triacontahedron (the other two patterns are their mirror images). The fat rhombs are the nine blue faces, and always separate into three "islands", either three groups of three (symmetrical decomposition) or one large five star island plus small islands of three and one.

It turns out all six Wayne Daniel puzzles use the asymmetric decomposition. In order to distinguish the asymmetric decomposition from its mirror image (Asymm. M), the final step is to locate the N and S poles, which requires taking the puzzle apart. Interestingly, although this analysis is necessary for understand the complete internal structure of the puzzle, it does not seem helpful for taking it apart!

These puzzles are more varied than the icosahedra puzzles. Not only do they range from 5 to 8 pieces, but the pieces themselves can be made from 5 to 9 half-rhombs. For some puzzles one piece comes out at a time; other times the assembly comes apart in unequal halves. The 7-piece and 8-piece puzzles seem a bit loose, and the 7-piece in particular can be disassembled by gently spinning it, the cracks which appear show how it likes to separate into unequal halves.

ID	First piece	Pieces	Woods Used	Decomp	3D print colour
WDRT50	Scorpion A	5	Peroba Rosa	Asymm	yellow
WDRT51	Scorpion B	5	unknown	Asymm	orange
WDRT52	Tarantula	6	unknown	Asymm M	red
WDRT53	Snake	6	Padauk	Asymm M	blue
WDRT54	7-Piece	7	Red Heart	Asymm M	black
WDRT55	8-Piece	8	Chechen	Asymm M	brown

Table 2. The six Wayne Daniel Triacontahedron puzzles.

If you want to identify a puzzle you find, first count the number of pieces. The two 6-piece puzzles can be easily distinguished by the shape of the first piece to come out (Tarantula or Snake). In order to distinguish Scorpion A and B, take note of the second piece to come out (after the "Scorpions", which are nearly identical). For Scorpion A this second piece is split 4/4, for Scorpion B, 3/5.

We should mention that Stewart Coffin has several puzzles in the shape of a triacontahedron, most notably his *Design No. 72*. This puzzle is quite different from those in this article because it has not been decomposed into rhombs. The pieces are instead created by drawing radial lines from the centre to each vertex, analogous to Coffin's *Garnet* puzzle (no. 60).

Summary

One feature of the Wayne Daniel split-rhomb puzzles is that they appear incredibly complex and intimidating, yet they are much easier than they appear. Disassembly can be problematic if the fit is very tight, but is usually relatively easy. If you begin from a pile of pieces, assembly may appear daunting, but if you press onward, there are actually very few ways the pieces can go together. Of course, the puzzles with more pieces (the triacontahedra) tend to be more difficult.

The authors wish to thank Wayne Daniel for creating these fascinating split-rhomb puzzles. We also thank James Dalgety, Stefan Garcia, and Stan Isaacs for loaning us puzzles or sending photos of their collection. We are reasonably confident that Table 1 is complete. We are less certain about the six triacontahedra. Please let us know if you find a Wayne Daniel triacontahedron puzzle which does not seem to match any in Table 2. We will add them in the supplementary material.

References

[1] G Bell and S Chin, Rhombic Polyhedra Puzzles, Part 1, CFF 109 (July 2019) pp 16-21.
[2] Zometool, http://www.zometool.com/
[3] A Gübeli, http://www.albinegri.ch
[4] http://puzzlemuseum.org