## Fifteen Puzzle


a.k.a. Game of Fifteen, Sliding Numbers, Gem Puzzle, Boss Puzzle, Le Taquin, ...

Old idea dating back at least to circa 1880, this version copyright ThinkFun 2000.
(metal with plastic case, 2.5 inches; keychain 1.75 inches)
Packaged with 1 through 15 arranged by row (lower right empty). After sliding pieces to mix it up, one must return to the starting position. The back of the box says that this one reproduces a 1933 design called the IMP:


To solve, the top two rows are easy, then cycle the 7 pieces on the last two rows, taking "short cuts" as needed to rearrange the order of pieces in the cycle.

## Other Fifteen Problems

Here are some other problems from the back of the ThinkFun Mini Fifteen keychain. The first, to make a magic square with the empty square counting as 0, is the Spanish Dungeon of H.E. Dudeney 1917 (see Baxter's Page). The last, the reversing problem, is noted as impossible.


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## The Fifteen-Fourteen Problem

A parity argument implies that half the puzzle configurations cannot be reached from any given configuration. For example, the starting position of the Fifteen-Fourteen puzzle shown on the left below has 14 and 15 exchanged, making the standard solution impossible (although as shown on the right below, solution is possible with the empty square in the upper right).


Fifteen-Fourteen, used by J. A. Storer as a child circa 1965. (plastic, 2.5 inches)
Here is a proof that the Fifteen-Fourteen problem cannot be solved, based on the presentation on the Wikipedia Page:

Define the count of a position to be the number of pairs of pieces that are out of order plus the number of the row that contains the empty square (rows are numbered 1 to 4 ). The parity of a position is even if its count is an even number and odd otherwise. Moving a piece one left or right does not change the position count since this does not change the ordering of pieces or the row number of the empty square. Moving a piece vertically always changes the position count by 4 because it changes the order with respect to three other pieces and changes the row number of the empty square by 1 . Hence, since both 0 and 4 are even numbers, each move preserves position parity, and all positions reachable from a given starting position must have the same parity. Thus, the 1-15 position cannot be reached from the 1-13-15-14 position because these positions have different parity.
In general, if you can get to where you have the position you want to reach except that in one place two adjacent squares are exchanged, then that position cannot be reached. For example, if someone gives you a what looks like a fifteen puzzle in a mixed up position, you can try to make the standard 1-15 position and either be successful or arrive at the 1-13-15-14 position (and be certain that this is a Fifteen-Fourteen puzzle for which a 1-15 solution is not possible). As another example, the reversing problem is not solvable, because it is possible to get to an almost reversed position except that the 1 and 2 are exchanged, as depicted below:


## Fifteen-Fourteen Problem Continued


"The Cross Number", circa early 1900's?
( $\operatorname{wood}$ box and 15 wood pieces, $4+3 / 8$ " x $4+3 / 8$ " x $1 / 2^{\prime \prime}$; directions on top specify a solvable version of the Fifteen-Fourteen problem)

## The Fifteen Magic Square Problem (a.k.a. 34 Problem)

Old versions of the fifteen puzzle typically had pieces that could be removed, and sometimes a piece 16 was included that was not used to play the normal fifteen puzzle, or left in for making a magic square of the numbers 1 to 16 , defined as an arrangement so that the four rows, the four columns, and the two diagonals all sum to 34 . One example is the Boss puzzle shown on the next page, which refers to this as the "Thirty Four" problem. Here is another:

"Le Taquin", manufactured by JJE Paris, circa 1880's.
(wood box and 16 wood pieces, 3.9 by 3.9 by $3 / 4$ inches; shown on page 61 of the Fifteen book, the French directions on the inside top cover describe both 15 and magic square)

The idea of magic squares dates to over a thousand years ago; here are two old designs shown on the Wikipedia page:

| 7 | 12 | 1 | 14 |
| :---: | :---: | :---: | :---: |
| 2 | 13 | 8 | 11 |
| 16 | 3 | 10 | 5 |
| 9 | 6 | 15 | 4 |


| 4 | 14 | 15 | 1 |
| :---: | :---: | :---: | :---: |
| 9 | 7 | 6 | 12 |
| 5 | 11 | 10 | 8 |
| 16 | 2 | 3 | 13 |

The Winning Ways book (page 778-783) discusses the design of $4 \times 4$ magic squares and notes that the 880 ways to do it for the numbers 1 to 16 (not counting reflections and rotations) was worked out in 1693 by Frenicle de Bessy; see also the Wolfram Mathworld page.

## Combined Puzzles

Many puzzles sold combined problems for fifteen and sixteen pieces; here are the front and back of directions that came with a " 2 puzzles in 1 " keychain puzzle:


INSTRUCTIONS FOR PLAYING THE " 16 " PUZZLE GAME
(A) When competing with a friend
I. Set both puzzle arrangements identically. Example - Spiral.
2. Select another arrangement.

Example - Perıpheral.
3. Start together - First to complete new arrangement wins.
(B) When playing alone
I. Check time for completing arrangements (average time - 15 minutes).
2. One to Sixteen (1-16), Sixteen to One (16-1), Odds first, Evens first, \& vertical are listed as impossible. CAN YOU DO THEM?
Note - 16 is molded or impressed in each vacant square.
(C) See examples on reverse side.
instructions for doing the "34" puzzle
A. The object of this puzzle is to arrange the numbers I thru 16 in such a manner that any group of four numbers horizontal, vertical, diagonal or in adjacent (grouped) position add up to " 34 " See diagram explanation on reverse side.
B. Note - 16 is impressed in each vacant square.
C. Use the 16 in the vacant squares as the key to various arrangements in which all groups of four numbers total " 34 " - as explained above.
D. See complete and incomplete sample arrangements on reverse side.
E. When sample arrangements are incomplete (numbers left out) the object is to fill in the missing numbers in order to make all totals of four numbers equal " 34 " vertically, horizontally, diagonally and in adjacent or grouped position.


SAMPLE ARRANGEMENTS OF THE "34"PUZZLE (THERE ARE MORE)

COMPLETE


| 15 | 4 | 5 | 10 |
| :--- | :--- | :--- | :--- |
|  |  |  |  |



ANY GROUP OF" 4 "NUMBERS ADD UP TO " 34 " DIAGONALLY VERTICALLY HORIZONTALIY OR ADJACENT

INCOMPLETE


FILL IM THE MISSING NUMBERS SO THAT AMY GROUP OF " 4 "TOTAL " 34 " DIAGOMALHY, VEKTICALLY, HORIZOITAUUYETC.

## The Boss 15 and 34 Puzzle


"BOSS THE NEW GAME OF FIFTEEN", W. H. Cremer, London, 1880. (cardboard box and 16 wood pieces, 3.5 by 3.5 by $5 / 8$ inches;
15 diagram on underside of the box top, and 17 page booklet about the 34 puzzle; similar to the puzzle shown on page 73 of the Fifteen book)

## Boss Boolet Pages About The Game Of 34

Messrs. Crumrn's Establishments are the Ingest of their kind in Europe; their stock includes the best ef everything in tho way of Toys, Dolls, Games, and Knick-knacks, select. ed with the utmost care; their large commands enatle them to offer superior articles at a more moverate coat than is usally the casc. Visitors who desire to expend bat a small amount will find a wonderful collection of pretty gifts, suitatle for all ages, at Cd. and lis. each.
Unlike the Establishments of France, Germany, and other countries (which, although admiralte, confine their sale generally to native production) Messrs. Caryms's stock embraces the manutactures of every clime and is most varkt ant orgemit in its charscter. A visit is respectully requested.

## tuI <br> "ALBRECHT DÜRER" <br> GAME of the THIRTY-FOUR.

This wonderfal combination of the number Thirty-four is over four husired years old, bence its interest as a novelty in psstime." It consists of 16 blocks numbered 1 to 16 . These are to be so arranged in the box that the addition sum of 34 masy be reekoned up in eighteen different ways, namely, perpendiculariy, horizontally, cross-wise from sorner to corner; by the four numbers forming esch group of four blocks at the cornees; by the four outside corner nambers; and by the two ontside num-

bers of the two centre lines at the top and bottom, also the two centre numbers on each side as shewn in the subjoined diagram:-

| 16 | 3 | 2 | 13 |
| :---: | :---: | :---: | :---: |
| 5 | 10 | 11 | 8 |
| 9 | 6 | 7 | 12 |
| 4 | 15 | 14 | 1 |

Aharecart Durer and Boss may be played either in company of ove or more, or as a Solitaire game: in either case these games will be found of no ordinary interest.

CREMER, Junlor,
210, Reorst St. axd 27, New Bosd St.
(Copyright.)

## "BOSS."

They wbo, from the present explanation, can learn how they may be able to move number for number in order to find an exact sequence, must feel deceived when we explain that only half the possible combinations can sucoeed.
Key to tie Boss Peztus:
Of the 15 blocks used in the game, 8 are with little tronble placod in the prescribed suocession, and so are four of the remaining i, either in a straight line, thus, $9.10 .11 . \overline{12}$, or in two columins :
9.10-11.12.
$13.14-14.15$, with the last three, which offer six combinations, this can be accomplished only in three cascs. The three remaining are absolutely insolvalle.

## 4

The six eombinations of the last three blocks are:

1. 2. 3. Eract Solution. Lowest. Mean. Higlest.

| 1 | 3. | 2. | Insolvable. |
| :---: | :---: | :---: | :---: |
| Low. | High. | Middle. |  |
| 2. | 1. | 3. | Insolvable. |
| Middle. | Low. | High. |  |
| 2. | 3. | 1. | Solvable. |
| Middle. High. | Low. |  |  |
| 3. | 1. | 2. | Solvable. |
| High. | Low. Middle. |  |  |
| 3. | 2. | 1. | Insolvable. |
| High. Middle. | Low. |  |  |

## 5

These combinations repeat themselves al. ways: if we begin with 1 and leave off with 12; or commence with 15 and end with 1 , we may first operate upon the side numbers, or the middle lines. With every combination which the 15 blocks allow, after entire permutation, the move terminates in one of the six comlinations of the last three blocks; and of these there remain for solution only the three before mentioned, but not the other three. With the absolute impossibility, through the removal of the blocks of the last two rows, to arrive at a solution, the game flnishes. If We try' by shifting the blocks already firmly set in the first two rows, we are launched in a new game. The possibility of a systematic

## Boss Puzzle Booklet Pages About The Game Of 34, Continued



11
We select at the outset the top and the bottom row:


We can test, with less trouble, by some irregular combinstion, the socuracy of our plan, and st the same time the practicability of the chosen combinations.

## Boss Puzzle Booklet Pages About The Game Of 34, Continued




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## An 1880 Newspaper Column On The Fifteen Puzzle

## This clipping was tucked into a copy of the 1893 Hoffmann book; from the text on the reverse side, it appears to be from a February 13, 1880 issue of an Albany newspaper.

## THE GAME OF FIFTEEN,

## A PUZZLE OVER WHICH PERSONS ARF SATD TO HAVE GONE CRAZY. <br> Solution of the Popular Puasle-The Myster

 ies of the Game of Fifteen Exposed.It is said that the game of "fifteen" was invented by N. P. Chapman, poalmaster of Canastots. Our readers, especially thoes who are wrecked on "imposeible combinations," will remember, that this is the same individual whose office was robbed of a pipe, pait of boots and other valuables no long since. The first one of the games invented found its way into the hands of a Syracuse lady, who gave it to an invalid lady at Wrteh Hill, R. I. This lady tonk it home to Hartford, Cunn., where it attracted considerable atteation. A Boston firm soon began manufacturin; it. It was not long before two firme were engaged in the manu facture of the blocks. Firm No. I tried to block the game of No. 2, but could not because the invention was not patented. The puzzte is being manufactured at the rate of 2,000 a day. The number of pissible combinations is said to be $1,397,674,368,000$, but there are bundreda of people who firmly belleve there ars ten times ss many ways in which it can't be done.

It is possible that the publication of a solution of the famous puzzle may interrupt rudely the reveriea of the phitosophers who have either solved the problem for themBelves or have the leisure to toil over its intricacies. But the conviction that \& solutlon will bear an olive branch of petcer to countlers strieken houveholds prompts the N. Y, Evening Poat in a spirit of broad philanthropy to offer what appeare to be une of several keys to the game.
For the sake of clearneas, says the Post, let us first agree that the row of numbers next to the stde of the box furtaest from the holder, and containing the numbers $1,2,3$ and 4 , shall be called A . The next parallel row, containing the numbers $5,6,7$ and 8 be called $B$; and the third row, contatuing the numbers $9,10,11$ and 12 be called C . We shad see that $B$ and $C$ are the important rows in our polution.
At the outset, instead of getting the lower numbers in their numerical onder, the quicker way to a solution is to arrange eleven numbers in their properiorder on the outside ross of the box. That order, it will be Been, is $1,2,3,4,8,12,15,14,13,9,5$, and We can begin with any one of these numbers and work either one way or both waye. This order csin be quickly secured by using the four central squares and three blocks. Move into the central equare the outer block (any one of thoee numbered above), turn it until opposite its proper place and make a vacancy for it by removing a block from cunther part of the outer rows and shifting the outer blocks around. So easily is this done that we leave it la the ingenuity of our readers without further instructions.

The outer blocks having been arranged the puzzle is limited to the four centra squares and the numbers $6,7,10$ and 11. which will rarely come in their proner order. These four numbera are susceptible of 24 combinations, whioh, however, as we shall show hereafter, may ba recolved iuto two, namely, it double inversion (in row $B$ 7. 6 and in row C 11, 10, and a single inver. sien (row $B$ correct and 11, 10 in row C). The double inversion is solved as followe, underatandiog the mandatory word move underatanding the mandatory word moy
before each of the numbers which foll-w:
$12,8,6,10,11,7,10,6,8,11,6,10,7,6$ $10,7,6,10,11,12$.

Let the foregciog formula be called X
The single inveraion (row B correct and 11,10 in row 0 ), which seems to involve the whole eecret of the puzzie, will be eolved by moving the following numbers in the order prescribed:
$12,8,7,10,11,6,10,7,8,11,7,10,6$, $7,11,8,10,11,7,6,11,10,8,7,10,11$ is $10,7,8,11,7,10,67,11,8,12$. We now have 7,11 in row $B$ and 6,10 in row $C$. Move all the outer blocks until number 5 is shified four places and is next to 6 Move the box one-quater around and it will be been that the solution is comple'e. Let the forezuing formula be calied Y.
We bave dieposed of three of the twenty. four combinations possible with the four central numbert. The three are: (1) The right order, (2) one double inversion and (3) one single inversion. Any one of the remaining twenty-oue combiustions can be quickly converted into one of the foregoing three by applying to them repeatedly the method of movea in formala X, or the method represented in the first eight moves of formuls X , or both methods combined. The moves are to be the same, though the order of numbers will, of course, be different Formula X, and also its tlet eight moves should therefore be practiced carefully before any conversion of the twenty-one com binations is tried. After the duuble or single inversion is obtained thes can then be solved by formulas X and Y respectively All this, which appears complex and difi cult on paper, will be readily underntood when the box can be used for illustration. The time for solving the puzzle from the beginning is from four to ten minutes.
Any possible combination of the numbers can, in the way we have explained, be therged into eome one of the four combinations of the four central blocks, and then solved by the formulss given. This applies of course, to the final comblaations of 15,14 , 13 , and $13,15,14$, sud $14,13,15$, which liave been 80 trying to the nerves and tempers of our provincial friends in Rochester.

There is snother story that a deafrmute in Hartford, Conn., invented this notorious game of fifteen. This is the slory current among those engrged in the manufacture of the puzzle. He told his friends about it, and they eaw more than a game in it, looking upon it as a mathematical study and its solution as a science. In a short time a factory was started to mske the puzzle for public sale. The N. Y, Sun eaye:
"A Broadway firm of dealers in five and ten cent goods sold 230 groes of this puzzle in one day. A member of the firm invented a new form of this puzzle:

$$
\begin{array}{llll}
\mathrm{L} & 1 & \mathrm{~T} & T \\
\mathrm{~L} & \mathrm{~B} & \mathrm{~B} & \mathrm{U} \\
\mathrm{~T} & \mathrm{~T} & \mathrm{E} & \mathrm{~B} \\
\mathrm{C} & \mathrm{U} & \mathrm{P} &
\end{array}
$$

He sent it out to dealers who ordered assortments of cheap goode, Nothing was heard from it for a time. Suddenly from Chieago, in holiday time, came a flood of orders The numerical puzzle was not yet knownthat ie, not yet in vogne there. The puzzle is utilized for advertising. A Southern transportation company has ordered boxes of blocks, which, when properly arranged, will spell the name of the company. A sew ing machine company has utlized it in the same manner. It is said that an up-town firm of jewellers has ordered 1,000 of these puzztes made of ivory blocks in fancy boxes. They are to retail for \$4 apiece. Another firm is baving the puzzle made in glass with the numbers ground upon the faces of the blocks. These are to sell for 25 cents,

By adding s sixteenth block, the game of thirty-four or the game of sixteen is produced. The ebject of the player is to 80 arrange the blocks that the snm of their numbers will be thirty-four when added horizontally, perpendienlarly, or diagonally The blocks may be taken out and changed in whatever manner the player phoopes. This is the sotation;

| 1 | 15 | 14 | 4 |
| ---: | ---: | ---: | ---: |
| 12 | 6 | 7 | 9 |
| 8 | 10 | 11 | 5 |
| 13 | 3 | 2 | 16 |

"The puzzle is not new. On the library ceiling in what is called the "castle," at the Schuelzen Park on Bergen Heighte, the puzzle is in freaco. It is over the hend of whoever lies on the lounge near the library window, and it is a favorite amusement of visitors there to lie and study it. Not only fis the sum of thirty-four arrived at in ail the lines diagonal, as horlzontal and perpendicular, but thirty-four is also the sam of each of the sets of four numbers compoaing the four corners, of the comner numbers themselves, sud, in fact, of every fout num bers that form smaller squares with the main square."

## IMP Puzzle - On Which the ThinkFun Version Was Based

Shown on page 102 of the Fifteen book. This 2.5 inch square metal puzzle was made in the 1933 to 1934 time frame in a number of similar variations, including different pegs on which the pieces slide (round vs. square), different colors, different text on the sides of the puzzle, different cases (shiny vs. textured red), similar but different booklets (all are 2.25 inches square with the same cover graphics), and even a braille version.

round pegs with black and white tiles, bottom edge says MADE IN U.S.A., left edge says
"IMP" PAT. APPLIED FOR, right edge says MODERN BRANDS INC. N. Y., top edge is blank

square pegs with black and red tiles, bottom edge says
"IMP" PAT. APPLIED FOR MADE IN U.S.A., top edge says
MODERN BRANDS INC. N. Y., other edges are blank

square pegs with red and white tiles, bottom edge says
"IMP" PAT. APPLIED FOR, top edge says
IMPORTED BRANDS INC. N. Y., left and right edges are blank

square pegs with black and white tiles, bottom edge says
"IMP" PAT. APPLIED FOR MADE IN U.S.A., top edge says MODERN BRANDS INC. N. Y., other edges are blank

square pegs with black and red tiles, bottom edge says
"IMP" PAT. APPLIED FOR, top edge says
IMPORTED BRANDS INC. N. Y., left and right edges are blank

square pegs with blue and white tiles, bottom edge says
"IMP" PAT. APPLIED FOR, top edge says
IMPORTED BRANDS INC. N. Y., left and right edges are blank

## IMP 1934 Booklet - Modern Brands

(from the black \& white round peg MODERN BRANDS version shown above)


## IMP*

- I am IMP, the world's most baffling number game. I am descended from the old "Fifteen" puzzle. Today I am internationally famous. Abroad they call me Diablotin. I am the rage they call me Diablotin. I am the rage Madrid, and Cairo. Famous People Madrid, and Cairo. Famous people play with me, Children delight in my antics. Shut-ins bless me for keeping them happily occupied. I have Chinese and Turkish cousins and a brother in Braille.
- I have over a trillion numerical combinations to test your skill and patience. Try me when you are lonely -test me in competition with your friends-play me any time, any place. I won't wear out and I promise to keep you entertained.
- To start your IMPing slide my numbers around until you have duplicated the sixteen arrangements illustrated on this side of the folder. After you have completed them see if you can solve the problems on the back of this sheet. Try to duplicate combinations marked $A$ and $B$ in each problem. You will be surprised to find that one of the arrangements is impossible. The answers will tell you if you can't find out.
*Trade Mark Reg. U. S. Pat. Oflice

| fime 16 is |  |  |  |  |  |  | Natanso |  |  | Oex ht |  |  |  |  |  |  |  |  |  |  | Aenter 16 is 13 term |  |  |  | 1wises |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 1 | 5 | 913 | 12 | 2 | 115 | 1 | 3 | 2 |  | 1 | 2 | 15 | 1 | 2 | 4 | 3 | 4 | 3 | 2 | 1 | 1 | 2 | 3 |  |
| 5 | 6 | 7 | 8 | 2 | 61 | 1014 | 7 | 9 | 104 | 5 | 7 | 6 | 8 | 3 | 4 | 1314 |  | 9 | II | 6 | 8 | 7 | 6 | 5 | 15 | 14 | 13 | 4 |
| 9 | 10 | 11 | 12 | 3 | 71 | 1115 | 11 | 5 | 68 | 9 | II | 10 | 12 | 5 | 6 | 12 II | 14 | 13 | 15 | 8 | 12 | II | 10 | 9 | 10 | 11 | 12 | 5 |
| 13 | 14 |  |  |  | 81 |  |  |  | 13.3 | 13 | 15 | 14 |  | 7 | 8 | 910 | 5 | 12 | 10 | 7 |  | 13 | 14 | 15 | 9 | 8 | 7 | 6 |
|  |  |  |  |  |  |  | '0is.antom |  |  |  |  |  |  | usperter |  |  | 2aters |  |  |  |  |  |  |  | Cowne |  |  |  |
|  | 12 | 8 | 4 | 2 | 46 | 68 | 4 | 5 | 1213 | 7 | 4 | 2 | 1 | 13 | 12 | 1110 | 2 | 4 | 1 | 3 | $4$ | $5$ | $6$ | 7 |  | 9 | 8 | 1 |
| 15 | 11 | 7 | 3 |  | 121 | 14 | 3 | 6 | 1114 | 11 | 8 | 5 | 3 | 14 | 3 | 49 | 6 | 8 | 5 | 7 | 3 | 2 | 1 | 8 | 15 | 10 | 7 | 2 |
| 14 | 10 | 6 | 2 | 1 | 35 | 57 | 2 | 7 | 1015 | 14 | 12 | 9 | 6 | 15 | 2 | 58 | 10 | 12 | 9 | 11 |  | 15 |  | 9 | 14 | 11 | 6 | 3 |
| 13 | 9 | 5 | 1 | 9 | II 1 | 1315 | 1 | 8 | 9 |  | 15 | 13 | 10 |  | 1 | 67 | 14 |  | 13 | 15 |  | 12 | II | 10 | 13 | 12 | 5 | 4 |



Copyright J. A. Storer

## IMP 1933 Booklet - Modern Brands

(from the black \& white square peg MODERN BRANDS version shown above)


## IMP 1933 Booklet - Party Bridge Play Inc.

(from the black \& red square peg IMPORTED BRANDS version shown above)


## I M P

$\star$ The Ultimate in Puzzle-Games

- The most fascinating-and baffling puzzle game ever invented.
- There are over a TRILLION possible combinations of the numbers 1 to 15 which should give you a fair idea of how much enterfainment awaits you as the owner of an IMP.
(C) 1933 P.B.P., Inc. Reproduction in whole or in part expressly forbidden.
- IMP is the greatest amusement value of all time-as it would take more than TWO MILLION years to solve all the problems-if you completed a new combination every minute.
- In playing with IMP the idea is first to try to duplicate all the possible problems in this booklet (the impossible ones are for sceptics and may be used to baffle your friends.)
- When you've worked out all the problems in the booklet, you're ready to have some real fun with IMP. Write the numbers 1 to 15 in any combination on a sheet of paper-then try to duplicate it on the game-you'll never know until you try whether it's a possible or an impossible problem.
- Arrange an IMP party (they're the rage in London, Paris, Berlin, Rome,


|  | anciminw | $=$ | $\underline{12}$ | Homum | - | митовия |  |  |  |  |  | urosatu |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 124 | 78910 <br> 8 | 151413 | 4 3 2 1 <br> 8    | 123 |  | 14 | 312 |  | 98 |  | 1 | 8 | 9 | 1 | 24 | 7 |
| 3 4 13 14 | 9116 | 615 11 | 9101112 | 8765 |  | 1 | 10 |  | 11 |  | 6 | 2 | 7 | 1015 | 3 | 5 | 11 |
|  |  |  | 5 678 <br>   | 1211109 |  | 7 | 6 | 54 | 12 | 1314 | 5 | 3 | 6 | 1114 | 6 | 912 | 214 |
| 78910 |  | 43211 |  | 131415 | 9876 |  | 2 | 1 | 1 | 23 | 4 | 4 | 5 | 1213 |  | 1315 |  |
| $\xrightarrow{-3}$ | T-m |  |  | =-20 |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | 1357 | 7 6 5 4 <br> 8    | 4567 | 981 |  | 14 | 1 | 4 | 32 | 1 |  | 13 | 119 |  | 12 | 3 |
| 14 3 4 9 <br> 15    | 6857 | 810129 | 815143 | 3 2 1 8 | 151072 |  | 12 | 84 | 5 | 1413 | 12 | 7 | 5 | 31 | 4 | 5 | 7 |
|  | 1012911 | 61411 | $9 \quad 132$ | 14159 | 141163 |  | 9 | 52 | 6 | 15 | 11 |  |  | 108 | 8 | 910 | 11 |
| 167 | 14. 1315 | 421513 | 1011121 | 13/12 1110 | 13 <br> 12 $\mathbf{5} 4$ |  | 6 | 1 | 7 |  |  |  | 4 |  |  | 131 | 415 |

## IMP Undated Booklet - Imported Brands Inc.

(from the blue \& white square peg IMPORTED BRANDS version shown above)

- All Europe is wild about this amazing new game. Millions of IMPS have been sold in Europe in the last few months. London is giving IMP parties, where all start a given problem at the same moment, and the first one to solve it gets the prize. In Berlin - Vienna Rome - Cairo - along the Riviera you see the IMP everywherel

Patent applied for
IMPORTED BRANDS, Inc. New York City


## I M P

- The most Fascinating-and baffling-puzzle-game ever invented!
- Gives a whole lifetime of amusement - for there are billions of different problems (some very easy, some very hard, and some-impossible) and by solving a new combination every minute you would be occupied for more than two million years!
- Never a bored moment, wherever you are-if you have an IMP in your pocket or purse. You can play it anywhere, alone or in company - and it always becomes the centre of interest!
- All pieces are locked in-hence can never be mislaid or lost. IMP is built with scientific precision to last for years I
- A few of the many problems are given in this folder. Try a few! You'll find that every friend, and every member of your family wants an IMP tool




## Graphic Versions of the Fifteen Puzzle

Many fun and promotional versions of the Fifteen puzzle have been made with graphics of some kind rather than numbers. Sometimes the graphics are such that every square is unique, and so it is really exactly the same puzzle as the standard Fifteen. However, when there are two pieces that are identical, as is the case with each of the four Warner Brothers puzzles shown below, it is possible to be stuck at a configuration where the puzzle is finished except that two adjacent pieces are out of order. In this case, solve the puzzle with the positions of the two identical pieces exchanged. For example, for the bugs bunny puzzle shown below, the pieces that go in positions 5 and 9 are identical (note that this is not the case for positions 8 and 12 because piece 12 is not quite blank); if you are not able to complete the last two rows because of this problem, move the blank piece that appears to go in position 9 to position 5 (causing the blank piece that was in position 5 to now be in the last two rows), and now solve the last two rows.


Bugs Bunny, Warner B. 1979. (plastic, 4.8 by 3.9 inches)


Road Runner, Warner B. 1979. (plastic, 4.8 by 3.9 inches)


Bugs Bunny / Daffy Duck, Warner B. 1979. (plastic, 4.8 by 3.9 inches)


Tweety, Warner B. 1979. (plastic, 4.8 by 3.9 inches)

## Roalex Versions of the Fifteen Puzzle


"Digit", Roalex Co., Forest Hills, NJ, circa 1950's and 1960's.
(card is 4.4 by 5.6 inches, puzzle is plastic 2.5 inches by $1 / 4$ inch thick)
A similar puzzle was made by Plastrix.

Roalex Versions of the Fifteen Puzzle, Continued

"Popeye", "Superman", "Yogi Bear", "Pebbles",
Roalex Co., Forest Hills, NJ, circa 1950's and 1960's. (card is 5 by 6 inches, puzzle is plastic 2.5 inches by $1 / 4$ inch thick)
The Roalex Co. made numerous Fifteen puzzles based on cartoons and TV shows; some based four related characters in each of the columns (such as the Popeye puzzle above) and some on individual characters (such as the Superman puzzle above that J. Storer played with as a child). These puzzles on their original cards (which sometimes had an extra piece on top) are a popular for collectors (see further reading).

## Roalex Versions of the Fifteen Puzzle, Continued



Roalex Co., Forest Hills, NJ, circa 1950's and 1960's. (card is 6.1 by 5.3 inches, puzzle is plastic 2.5 inches by $1 / 4$ inch thick)
Although many Roalex cards were horizontal ones of an approximate shape of width 1.25 times height as with the version of Popeye on the preceding page, other shapes were used, including a shape of about height 1.14 times width, such as this version of Popeye.

## Roalex Versions of the Fifteen Puzzle, Continued

Here is what is on the back of the Popeye card of the preceding page:


## Other Versions of the Fifteen Puzzle


"Gem Puzzle No. 0", Matthias Rice, December, 1879.
(3.25 inches square by $1 / 2$ inch thick cardboard box and 15 wood pieces; shown on the cover page, page 8, and page 11 of the Fifteen book, which dates this puzzle and gives some history)

The top of the box top says "THE GEM PUZZLE No. 0", the bottom of the box top says "Place the Blocks in the Box irregularly, then move until in regular order.", the left and right sides have been scratched out on this one, but originally on the left side was "MANUFACTURED BY M. J. RICE" and on the right side "For CARY, FULTON \& Co., No. 29 Kingston Street, Boston."
Although the theme of the Fifteen book is that the origin of the Fifteen puzzle is unknown, it does indicate that the high popularity of the puzzle in the 1880 time frame started with this production of the puzzle in December of 1879, and describes a March 1, 1880 interview of Mr. Rice published in the Boston Herald that describes how he got the idea for making the puzzle from a version made in Hartford by deaf students, and sold for 75 cents apiece.

Other Versions of the Fifteen Puzzle, Continued

"Drueke's 15 Puzzle", Wm. F. Drueke \& Sons, Grand Rapids, Mich.", circa 1960's. (plastic, 2.5 inches square by $3 / 16$ inch thick)

## Other Versions of the Fifteen Puzzle, Continued


"15-Puzzle", Rudolph Steiner, NY, circa 1950's.
(cloth pouch, plastic puzzle, and cardboard instructions, 2.5 inches;
the back says THE "15-PUZZLE", ARRANGE NUMBERS. HORIZONTALLY, VERTICALLY, DIAGONALLY, OR IN SPIRALS, ETC., PAT. APPLIED FOR, RUDOLPH STEINER CO., N.Y.C. U.S.A.)

"Lowe's 15-Puzzle", circa 1950's.
(felt lined pouch, plastic puzzle, and cardboard instructions, 2.5 inches; shown on page 103 of the Fifteen book)

## Other Versions of the Fifteen Puzzle, Continued




Vol. 515
INSTRUCTIONS
 15 Puzzle


27 W. 20th St., K. Y. C.

There are 17 other volumes of fun in the popular


Keep adding to your collection regularly. In only a short time you will own the entire set . . . 18 good reasons why there will never be a dull moment in your home or when you travel. Perfect gifts for men in the service.

## Have Jun!

"15 Puzzle", Lowe's, circa 1940?
(4.75" square by $1^{\prime \prime}$ leather covered box with fifteen $1^{\prime \prime}$ square by $1 / 2^{\prime \prime}$ wood pieces)

## Other Versions of the Fifteen Puzzle, Continued



| HOW TO SET TO WORK. <br> First of all arrange the small squares in the order of the diagram on the left. Take out and put aside the piece numbered 16 and then slide (not lift) the remaining pieces until you have arranged them in the order of the diagram on the right, after replacing 16 in the lank space and cumpleting the Puzzle. <br> HOW TO START <br> HOW TO FINISH |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 16 | 15 | 14 | 13 | 1 | 2 | 3 | 4 |
| 12 | 11 | 10 | 9 | 5 | 6 | 7 | 8 |
| 8 | 7 | 6 | 5 | 9 | 10 | 11 | 12 |
| 4 | 3 | 2 | 1 | 13 | 14 | 15 | 16 |

"Great 16 Puzzle", Tit-Bits Teaser No. 6, circa 1930's.
(wood pieces in cardboard box, $3.6^{\prime \prime}$ square x $3 / 8 "$ thick)

## Other Versions of the Fifteen Puzzle, Continued


"Celebrated Fifteen Puzzle", Fairy Queen Steamer, circa late 1800's? (wood pieces in hinged wood box, $2.3^{\prime \prime}$ square x $3 / 4^{\prime \prime}$ thick)

The Fairy Queen steamer boat is mentioned in the 1885 Thorough Guide Series for Scotland; here is an excerpt from page 185:

Hoch Eck Route. From Dunoon the coach take the coast-route throngh Kirn, Hunter's Quay-leafy and Iuxuriant-and Sandoank, whence it proceeds round the head of the Holy Loch into the Echaig valley, whose waters connect Loch Eck with the sea. At Inverchapel the passengers are transferred to the ittle steamer "Fairy Queen," which conveys them the whole length of the loch.

## Other Versions of the Fifteen Puzzle, Continued



Wood box with inlay of dancing couple and 15 wood pieces, 1837 ??? (4.6 inches square by 1 inch thick, pieces are 1 inch square by $1 / 4$ inch thick, the date 1837 is hand written on the back)
This box has a beautiful inlaid top showing a dancing couple and looks quite old. The date of 1837 written on the back raises the fun possibility that this puzzle pre-dates the 1880's Fifteen puzzle craze that is documented in the Fifteen book. However, it is hard to give this date too much weight; it could have been written by anyone at anytime. Below are photos of the inside, which looks quite similar (including the hinges) to the inside of the Souvenir d'Egypt puzzle (made in France) that is shown on page 97 of the Fifteen book.


## Other Versions of the Fifteen Puzzle, Continued


"Calculator Puzzles", England, circa 1880's.
(4.5" square by $7 / 8^{\prime \prime}$ wood box and sixteen $1^{\prime \prime}$ diameter by $3 / 8^{\prime \prime}$ round painted wood pieces;
paper label on box top and rules on underside of box top;
same box as the one shown on page 25 of the Fifteen book)

## Other Versions of the Fifteen Puzzle, Continued


"Little Buttercup Puzzle", B. F. Gould, 40 Bromfield St., Boston, 1880. (cardboard box and 15 wood pieces, 3 by 3 by 1 inch;
the ridged tops have the numbers 1-15 and the smooth bottoms have letters (close-ups of the piece P/5 are shown above),
the directions on the box top ask you to spell LITTLEBUTTERCUP
(the fourth T and the C are a too worn to read in the photo above), the Fifteen book shows this puzzle on pages 20, 36, and 49
where it credits manufacture to B. F. Gould and shows a Feb. 1880 advertisement)


Hopkins, Oxford, NY, circa 1880's.
(wood tray and 15 wood pieces, 3.7 inches;
1 is a bit burned, 5 is damaged, and 10 was lost and replaced by a blank, the back is stamped "J. A. Hopkins MAKER Oxford NY",
from J. A. Storer's grandfather in Oxford NY)

## Other Versions of the Fifteen Puzzle, Continued


"The Game of Fifteen Gem Puzzle", manufactured by Alan L. Lovejoy, Boston, 1880. (cardboard box. wood tray, and 15 wood pieces, 3.75 by 3.75 by $3 / 4$ inches; shown on page 19 of the Fifteen book where it cites manufacture and date)

"The Game of Fifteen Gem Puzzle", circa 1880.
(cardboard box. wood tray, and 15 wood pieces, 3.75 by 3.75 by $3 / 4$ inches; shown on page 23 of the Fifteen book; box says "SENT TO ANY ADDRESS FOR 25 CENTS".)

## Other Versions of the Fifteen Puzzle, Continued


"Double Puzzle Of Crack Brain And Thirty Four", Heyer Brothers, Boston, circa 1880. (cardboard box, wood tray, and 16 wood pieces, 3.9 by 3.9 by $3 / 4$ inches;
directions on the inside of the box top;
shown on page 40 of the Fifteen book)

## Other Versions of the Fifteen Puzzle, Continued


"The Gem Puzzle / Double Puzzle", circa 1880's.
(cardboard box and 16 wood pieces, 3.25 inches square by $9 / 16$ inches; shown on page 34 of the Fifteen book;
includes piece 16 to have the magic square as a second puzzle)

Other Versions of the Fifteen Puzzle, Continued

"The Boston Puzzle", circa 1880's.
(cardboard box and 15 wood pieces, 3 inches square by $5 / 8$ inches; different than the "Boston Puzzle" shown on page 24 of the Fifteen book)

Other Versions of the Fifteen Puzzle, Continued


## The Old "Fifteen Puzzle"

If a new puzzle is needed to take the place- of cross-words, why not revive the old "Fifteen Puzzle?" This "teaser" flourished in the days when Hayes was president. At that time a well known weekly paper had a cartoon showing President Hayes and all the cabinet members working a Fifteen Puzzle at a cabinet meeting. This puzzle consisted of a little box, the bottom of which was marked off in 16 squares. There were 15 cubes, numbered from one to 15. Cube No. 1 was placed in square No. 1 in the left hand corner of the box. All the cubes were placed in regular order until the top row was
jaxtany sa, 1208 ion Hour ~
reached. Here, instead of placing them in order, $13,14,15$, they were reversed. The 15 was at the left of the row and they read $15,14,13$, and the 16 th square was left vacant. The game was to move the cubes around from one square to another until you had them all straight, reading from one to 15 . These cubes could not be lifted from the box in moving them. The boxes were of pine or ebony with cubes to match. On some expensive sets the numbers on the ebony were inlaid with mother-of-pearl. If you want to have a "puzziing sweet time" try to work a "Fifteen Puzzle."
"The Puzzle Of 15 and 16", circa 1880's.
(cardboard box and 16 wood pieces, 3.25 inches square by $51 / 2$ inches; shown on page 38 of the Fifteen book;
"This little puzzle looks simple and easy but TRY IT ONCE."; this one came with an article from a 1926 newspaper that reflects on the Fifteen Puzzle as something from the past when R.B. Hayes was president)

## Other Versions of the Fifteen Puzzle, Continued


"The Popular Fifteen Puzzle", F. Passmore, London, circa 1880's. (cardboard box and 15 wood pieces, 4.2 inches square by $5 / 8$ inches; directions on the inside of the box top;
Shown on page 30 of the Fifteen book, but listed with a different English manufacture;
a very similar box top is also shown inside the cover of the Fifteen book)

Other Versions of the Fifteen Puzzle, Continued


Die Steine sind durch Vorwärtsund Rückwärtsschieben in die Reihenfolge von 1-15 zu bringen.

Blocks are brought by moving forwards and backwards in a successive rou from 1 to 15 .

Essayez en mouvant les pièces en avant et en arrière de mettre une file successive de 1 à 15 .

German, circa 1880's.
(cardboard box and wood pieces, $2.5 \times 2.5 \times 3 / 8$ inches; shown on page 121 of the Fifteen book)

On the page of the Fifteen book that shows this puzzle is a nice discussion of how newspapers from February and March of 1880 had a large number of "notes, articles, and poems that claimed that the Fifteen Puzzle was driving solvers insane and overcrowding the lunatic asylums".


## 15 Puzzle.

This game of patience is of great antiquity and is played as follows: -

The Counters are laid in the box after having been well shuffled so that they do not run in numerical order. The object of the Game is to move them into their proper sequence without removing any counter from the box, but merely by sliding them about in the box to their correct positions. (SPEAR'S GAMES)

Made at the Spear Works Bavaris.
"15 Puzzle", Spear Works Bavaria 1915.
(cardboard box and wood pieces, $4 \times 4 \times 5 / 8$ inches;
shown on page 119 of the Fifteen book where it cites manufacture and date)

## Other Versions of the Fifteen Puzzle, Continued



The "Thirty Four" Puzzle consists of 16 blocks, numbered from 1 to 16 , these
The "Fifteen" Puzzle consists of the are in a small square box, and the Puzzle same blooks or figures, with the figure 16 is to so place the figures as to add up in taken out as on the plan; leaving whu 16 different ways, each to make Thirty blank space so as to give room to muve Four, the addition must be the same the Blocks. After well mixing tho lig counting from corner to corner, horizonures place them in the box, and muvo tally, perpendicularly, and the four blocks them without taking them out until they in each corner, also tho four outside numare numerically in order. bers of the two middle lines.
"Gem Puzzle" by John Heywood, Manchester, UK, undated. (cardboard box and 16 wood pieces, $3.4 \times 3.4 \times 1 / 2$ inch; shown on page 29 of the Fifteen book)

"15 and 34 puzzle", De La Rue \& Co., London, circa 1880. (cardboard box and 16 wood pieces, $3.75 \times 3.75 \times 5 / 8$ inch; shown on page 35 of the Fifteen book)

## Other Versions of the Fifteen Puzzle, Continued


"King George VI Coronation Puzzle", circa 1937.
(cardboard box and 16 cardboard pieces, $4.25 \times 4.25 \times 1 / 4$ inch; inside of box top has directions;
inside of the box bottom advertises Meadow Butter; both the puzzle pieces and the box top have photos of the royal family; to read about king George VI, see for example the Wikipedia Page)

## Other Versions of the Fifteen Puzzle, Continued


"Magic 16 Puzzle", Copyright the Embossing Company, Albany, NY, 1930.
( $3.3^{\prime \prime} \times 3.3^{\prime \prime} \times 9 / 16^{\prime \prime}$, sixteen $3 / 4^{\prime \prime}$ square by $1 / 2^{\prime \prime}$ thick wood pieces)

## Other Versions of the Fifteen Puzzle, Continued




CHALIENGE 2

| 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: |
| 6 | 1 | 2 | 11 |
| 5 | 4 | 3 | 12 |
| $\mathbf{z}$ | 15 | 14 | 13 |
| challenge 3 |  |  |  |


| 1 | 3 | 5 | 7 |
| :---: | :---: | :---: | :---: |
| 9 | 11 | 11 | 15 |
| 2 | 4 | 6 | 8 |
| 10 | 12 | 14 |  |

CHALLENGE 7


These are but a few of the porsible combinations-a mathematician tells us there are over one billion!-so get to work and discover some for youreclf. Always estaring from the Original Position, keep a record of them and see if you can solve them later on.

THE EMBOSSING COMPANY
ALBANY, N. Y.
"15 Puzzle", The Embossing Company, Albany, NY, circa 1937.
(cardboard box and 15 wood pieces, $4.2 \times 4.2 \times 5 / 8$ inch;
this red version appears to have a second 6 instead of a 9 , same manufacturer and box size / style as the Time and Missionary Puzzles)

## Other Versions of the Fifteen Puzzle, Continued


"The Combination Puzzling Puzzles", copyright Canada 1934.
(wood box, 15 wood pieces, 3.9 by 3.9 by $7 / 8$ inches;
flip the puzzle over and the backs of the pieces have the letters GDOAETYNANALNI?,
? for piece 13 that has been replaced and had A hand written on the back)

| 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| 5 | 6 | 7 | 8 |
| 9 | 10 | 11 | 12 |
| 13 | 14 | 15 |  |

Fig. 1


Fig. 2


Fig. 3
The object of the Fifteen Puzzle is to arrange the numbers in positions as shown on the Giagrams above. After you have worked these, you can create problems of your own. Merely draw any diagram or any combination of 15 numbers and follow them in the Puzzle. Place the Puzzle on a flat surface and slide the numbers with your finger or a rubber eraser, or plck up the Puzzle and slide the numbers by gravity.
No matter how fast you solve a problem, you can always improve your time or lessen the number of moves. It has infmite variety and you never do it the same way twice. Every time you do this puzsle, it becomes more interesting.
${ }^{*}{ }^{\circ}$ MANUFACTURED BY s. S. ADAMS CO , ASBURY PARK, N, d.

Adams Co., unknown age.
(cardboard case and metal puzzle, 3.25 inches)

## Other Versions of the Fifteen Puzzle, Continued - "Dukes Of Hazzard"



Dukes Of Hazzard, Warner B. 1981. (plastic, 4.8 by 3.9 inches)


Boss Hogg, Warner B. 1981. (plastic, 4.8 by 3.9 inches)


Bo, Luke, and Daisy Duke, Warner B. 1981. (plastic, 4.8 by 3.9 inches)


General Lee, Warner B. 1981. (plastic, 4.8 by 3.9 inches)

## Other Versions of the Fifteen Puzzle, Continued



Superman, D C Comics 1978. (plastic, 4.8 by 3.9 inches)


Spiderman, Marvel Comics 1978. (plastic, 4.8 by 3.9 inches)


Batman, D C Comics 1978.
(plastic, 4.8 by 3.9 inches)


Incredible Hulk, Marvel Comics 1978. (plastic, 4.8 by 3.9 inches)

## Other Versions of the Fifteen Puzzle, Continued



Snap Crackle Pop, Kellog Companys 1979. (plastic, 4.8 by 3.9 inches)


Toucan Sam, (c) Kellog Company 1979. (plastic, 4.7 by 3.8 inches)


Dig'Em Kellog Companys 1979. (plastic, 4.8 by 3.9 inches)


Popeye, King Features 1981. (plastic, 4.8 by 3.9 inches)

## Other Versions of the Fifteen Puzzle, Continued



Circa 1960's. (brass, 3.25 inches)


The Monitor, Artist Series, Philips, no date. (plastic, 3.5" x 2.9" x 1/4"; sticker on back shows solved position)


Marge \& Homer Simpson, circa 2000. (plastic, 2.5 inches)


Hungarian, circa 1950? (metal, 2.75 inches)


Alphabet, circa 1960's. (plastic, 2.5 inches)


Bart Simpson, circa 2000. (plastic, 2.5 inches)

## Other Versions of the Fifteen Puzzle, Continued



101 Dalmatians, Disney, circa 1960's? (plastic, 3.5 by 3 inches)


Donald Duck, Walt Disney Productions, circa 1960's? (plastic, $2+5 / 8^{\prime \prime} \times 2+5 / 8^{\prime \prime} \times 3 / 16 "$ )

## Other Versions of the Fifteen Puzzle, Continued



Santa Claus, circa 2000?
(plastic, $2+3 / 4^{\prime \prime} \times 4.5^{\prime \prime} \times 3 / 16^{\prime \prime}$ )

## Further reading:

Slocum's Page: http://www.puzzleworld.org/PuzzleWorld/jerry_slocum.htm
Baxter's Page: http://www.johnrausch.com/SlidingBlockPuzzles Jaap's Page, from: http://www.geocities.com/jaapsch/puzzles/fifteen.htm
Wikipedia Ffiteen Page, from: http://en.wikipedia.org/wiki/Fifteen_puzzle
Wikipedia Magic Square Page, from: http://en.wikipedia.org/wiki/Magic_square
Wolfram Magic Square Page, from: http://mathworld.wolfram.com/MagicSquare.html
May Patent, from: www.uspto.gov - patent no. 50,608
Kinsey Patent, from: www.uspto.gov - patent no. 207,124
McCleary Patent, from: www.uspto.gov - patent no. 284,037
Brown Patent, from: www.uspto.gov - patent no. 390,829
Bradshaw Patent, from: www.uspto.gov - patent no. 427,392
Brown Patent, from: www.uspto.gov - patent no. 433,444
Cook Patent, from: www.uspto.gov - patent no. 476,980
Anderson Patent, from: www.uspto.gov - patent no. 483,276
Eymann Patent, from: www.uspto.gov - patent no. 535,279
Johnson Patent, from: www.uspto.gov - patent no. 1,555,980
Fritz Patent, from: www.uspto.gov - patent no. 1,693,711
Nesis Patent, from: www.uspto.gov - patent no. 5,785,318

